Collective Households and the Limits to Redistribution*

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Abstract

This paper explores optimal distributive policies using a Collective approach to household behavior. This approach allows individual preferences for each spouse, which is crucial when examining policies targeted to women, like Mexico's *Prospera* or Brazil's *Bolsa Família* programs. We assume the spouses' decisions follow a Nash-bargaining procedure with internal threat points. We show that the taxation principle does not apply, meaning that the optimal tax schedule is dominated by the optimal mechanism. This is because a single tax schedule cannot optimally influence threat points and induce households to choose desired allocations simultaneously. By permitting couples to opt for joint or individual tax filing, we significantly increase the set of implementable allocations. The central role of threat points motivates an extension of the model in which marriage market negotiations partially determine threat points after marriage. This extension endogenizes the distribution of couples and formalizes how general equilibrium considerations and social norms affect bargaining power within a couple. The extended model generalizes previous approaches to the optimal taxation of couples. Finally, we parametrize and calibrate this model to empirically evaluate the relevance of our theoretical findings. The ability to influence threat points has large effects on equilibrium allocations and the evaluation of optimal policies. ***Keywords:*** *Mechanism design; Collective Households; Taxation Principle.* ***JEL Codes:*** *D13; H21; H31.*

1 Introduction

The 1979 UK Reform of Child Benefits is but one example of policy interventions that change the relative income of women versus men with the explicit purpose of empowering women. These reforms that transfer money 'from the pocket to the purse' are inspired by mounting evidence that the identity of the resource recipient affects household decisions —Thomas (1990); Browning et al. (1994);

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Browning and Chiappori (1998); Browning and Gørtz (2012); Duflo (2003); Almås et al. (2018); Armand et al. (2020). The same type of evidence informs the design of most current conditional cash transfer programs as *Prospera* in Mexico (originally *Progresa*, then *Oportunidades*) and *Bolsa Família* in Brazil.

This type of evidence rejecting income-pooling is not compatible with the *Unitary* approach to household behavior, under which multi-person households are treated as if they were individuals. Yet, in contrast with evidence and policy practice, only recently have the theoretical analyses of redistribution policies begun moving away from Unitary household models. And while important advances have occurred—Gayle and Shephard (2019)—the connection between evidence-based policy and theoretical approaches capable of exploring the limits of distributive policy is still wanting. The goal of our work is to fill the gap by replacing the unitary household model with a *Collective* model—Apps and Rees (1988); Chiappori (1988, 1992)—that is capable of rationalizing the evidence while retaining enough structure to assess optimal policy in the Mirrlees' tradition.

We specialize the Collective model by building on Lundberg and Pollak's (1993) view of a household's decision process. Married persons are assumed to decide following a Nash-bargaining procedure with *internal* threat points: the *disagreement* utilities attained within the marriage but with spouses behaving non-cooperatively. This model is, in particular, compatible with the findings in Lundberg et al. (1997); Almås et al. (2018); Armand et al. (2020) for reform or policies that only change policy for married agents leaving intact the incentives for singlehood. As we show, it also allows us to use traditional mechanism design tools and explore the limits of distributive policies.

Before we describe our main findings, we must point to recent works that do take the multi-person nature of households seriously. They typically take for granted the set of policy instruments—e.g., Gayle and Shephard (2019)—or assume that policy cannot affect the distribution of power across spouses—Kleven et al. (2009); Alves et al. (2021); Golosov and Krasikov (2023). This paper, instead, lays the foundation for analyses of redistribution policies with multi-agent households in the Mirrlees' tradition. As in Mirrlees (1971), we use a direct mechanism to characterize the set of implementable allocations and consider when the allocation can be decentralized through suitably designed tax schedules, the taxation principle. We first show how to define the type for married couples so that the *revelation principle* remains valid in our setting. This allows us to provide a simple characterization for the set of incentive-feasible allocations. However, the *taxation principle* fails; not all incentive-feasible allocations can be decentralized with a budget set.

The taxation principle failure is due to the budget set playing two distinct roles. First, as usual, they induce choices conditional on households' objectives. Second, they affect these objectives through threat points, which are spouses' disagreement utilities when the couple behaves non-cooperatively. A single instrument for these two objectives makes decentralization with a single budget set sub-optimal. In particular, we show how adding the option to file either jointly or individually expands the set of implementable allocations relative to a unique tax schedule. This filing option is similar to the one already in place in the U.S. tax system. We also offer conditions under which the full set of incentive-feasible allocations can be implemented with filling options.

To simplify our exposition, we first present our main theoretical results assuming a fixed distribution of couples and ignoring the impact of redistribution policies on the marriage market. Perhaps more importantly, we disregard the impact of marriage market conditions on the distribution of power. We remedy this by expanding our model to include the marriage market. By doing so we endogenize the distribution of households and let the rules for the disagreement game within each household be partially chosen at this stage. While Lundberg and Pollak (1993) emphasizes the role of social norms for the disagreement game, our approach allows marriage market conditions to influence allocations in marriage. Our approach, therefore, generalizes both Pollak's (2019) Bargain-in-Marriage and Choo and Siow's (2006) and Gayle and Shephard's (2019) (imperfectly) transferable utility models.

One thing we rule out in our analysis is the extreme form of commitment (or contract completeness) that allows all conflicts to be resolved at the marriage market stage. Or taken from the opposite angle, while our approach allows for choices at the marriage market stage to affect threat points, post-marriage bargaining still plays a role.[1]

The rest of the paper is organized as follows. In Section 2, we present our model of household behavior under the assumption that the distribution of couples and the mapping from transactions to threat points is exogenous. Section 3 contains the main theoretical results of our paper. The household model is generalized in Section 4 to endogenize the role of marriage markets. In Section 5, we specify a particular parametric model and calibrate it to the U.S. economy to provide a quantitative assessment of the forces identified in our theoretical model. Section 6 concludes.

Literature Review

The main goal of this paper is to offer a framework for studying optimal distributive policies for multi-person households in the Mirrlees' tradition.

The literature on optimal taxation of multi-person households started with Boskin and Sheshinski (1983). They considered linear parametric taxes applied to a unitary household and, since labor earnings are assignable and women have higher labor elasticities, they derived optimal taxes that are lower for wives. Boskin and Sheshinski (1983) relied on a unitary framework and did not take into account that the household behavior need not have a normative content; Apps and Rees (1988) was the first to address this issue, which they called *dissonance*.

[1]As suggested by Lundberg and Pollak (1993), "The marriage market will wholly undo any redistributive effects if prospective couples can make binding, costlessly enforceable, prenuptial agreements to transfer resources within the marriage; dowry and bride price can, under certain circumstances, be interpreted as examples of practices that facilitate such Ricardian equivalence. If binding agreements cannot be made in the marriage market and we think this is the relevant case for advanced, industrial societies child allowances may have long-run distributional effects."

An extension of this literature has characterized optimal non-linear taxes for unitary households—e.g. Kleven et al. (2009); Immervoll et al. (2011); Cremer et al. (2016); Golosov and Krasikov (2023); and Alves et al. (2021). The main technical difficulty has been that modeling a multi-person household naturally leads to considering multi-dimensional types, which complicates the characterization of optimal policies within a Mirrlees' framework. To overcome this issue, Kleven et al. (2009) assumed that primary earners make intensive margin decisions while secondary earners only make extensive margin decisions, Immervoll et al. (2011) only considered extensive margin decisions, Cremer et al. (2016) used a stylized model with finite types, and Alves et al. (2021) considered only joint taxation and identical iso-elastic preferences. Golosov and Krasikov (2023) shows that the optimal tax schedule for couples solves a second-order differential equation and provides a characterization of this solution. They all abstract from the interaction between policy and the intra-household distribution of power.

Another extension has departed from the unitary framework and has considered the impact of policies on the power within households. To the best of our knowledge, Alesina et al. (2011), Bastani (2013) and Gayle and Shephard (2019) are the only attempts to characterize optimal taxes for Collective households—as defined by Browning et al. (2006). Alesina et al. (2011) and Bastani (2013) considered a Nash-bargaining household model with singles' utilities as threat points. They both assumed quasi-linear utilities and linear tax schedules; Alesina et al. (2011) abstracted from heterogeneous productivity, while Bastani (2013) assumed that spouses in a couple are equally productive. They both abstracted from marriage considerations.[2] In contrast, Gayle and Shephard (2019) assumed that threat points are defined by binding contracts that clear the market at the marriage stage: spouses can credibly promise to forgo power within the marriage to get better matches at the marriage market.

Differently from Gayle and Shephard (2019), we assume that not all disputes between spouses are solved at the marriage market stage. This form of contract incompleteness leads to the use of *internal threat points*, a feature of bargaining models that allows us to accommodate the empirical evidence in Armand et al. (2020) and to rationalize the design of income-support programs as *Bolsa Família* in Brazil and *Oportunidades* in Mexico, neither of which can be done with Gayle and Shephard's strong commitment assumption.

Internal threat points, the utilities attained by spouses when they fail to agree on a course of action, behaving non-cooperatively instead, were first proposed by Lundberg and Pollak (1993). While a growing empirical literature has supported the relevance of internal threat points—e.g. Mazzocco (2007) and Armand et al. (2020)—and a theoretical and experimental literature has justified it—e.g. Bergstrom (1996); Binmore (1985); Binmore et al. (1989)—our approach does not disregard the influence of marriage market conditions for the distribution of power between spouses. By assuming contract incompleteness at the marriage market stage, we encompass both the full commitment—Choo and Siow (2006); Gayle and Shephard (2019)—and the bargaining-in-marriage (BiM) approaches—

[2] Bastani (2013), in his quantitative analysis, relaxed the quasi-linearity assumption and considered internal threat points.

Pollak (2019)—as special cases of our approach.

As far as we know, our work is the first extension of mechanism design to Collective households. This extension allows us to discuss how auxiliary non-rate policies impact allocations, whereas the previously cited works, could not. In his seminal contribution, Mirrlees (1971) pioneered the idea that optimal distributive policies should be found by making explicit the information structure that precluded the attainment of first-best allocations. In the next few years, the notion of incentive compatibility—Hurwicz (1972)—and the revelation principle—Myerson (1979); Dasgupta et al. (1979); Harris and Townsend (1981)—provided the language for mechanism design. If on the one hand, the revelation principle allowed one to characterize the full set of incentive-feasible allocations thus establishing the limits to redistribution, the taxation principle—Hammond (1979); Guesnerie (1981)—guaranteed that a single budget set could decentralize these allocations. All these fundamental results were derived under the assumption that individuals are single or that multi-person households can be treated as if they were individuals; the so-called Unitary model. We prove a version of the revelation principle for our household model but find that the taxation principle no longer applies.

2 Model with Exogenous Marriage

Our framework builds on Lundberg and Pollak's (1993) approach to household decision-making, where spouses follow a bargaining protocol satisfying Nash's (1950) axioms, and threat points are the utilities from a non-cooperative Nash equilibrium within marriage. These *internal* threat points, as opposed to divorce threats, allow for policy changes to affect a spouse's bargaining power even if it is irrelevant to his or her utility as a divorcee, which is consistent with the empirical evidence—Lundberg et al. (1997); Almås et al. (2018); Armand et al. (2020).

In this section, we assume that the marriage distribution is given and unaffected by changes in policy.[3] Furthermore, the bargaining power in a household is only a function of their disagreement transactions and policy-invariant social norms; no *external* consideration, like the policy impact on other couples, affects the couple's decision.

With this assumption, we can prove our main results in the simplest setting. In Section 4 we extend the model to allow for policy to affect the marriage market and, in turn, for it to affect threat points. The main results from the current section survive with (essentially) identical proofs even if the marriage distribution is endogenous. Our extended model has the BiM, the Choo and Siow (2006), and the Gayle and Shephard (2019) models as special cases.

[3]We also abstract from single agents since it adds nothing to the results under the assumption of exogenous distributions.

2.1 Environment

Households have two identifiable spouses, $i = f, m$, with identical preferences over private consumption, c, and leisure, ℓ: $v(c, \ell)$.[4] Each spouse has an endowment of $(c = 0, \ell = 1)$, and is identified by their gender i and productivity $w_i \in W \subset (0, \infty)$. We denote the household with productivities w_f and w_m by the ordered pair $\boldsymbol{w} = (w_f, w_m) \in W \times W$. In the rest of the paper, we follow the convention that boldface variables refer to the household, while non-bold variables to an individual spouse. Under the current assumption of exogenous marriage, the fraction of $\boldsymbol{w}$ households is fixed and denoted by $\mu(\boldsymbol{w})$.

Each spouse provides labor z_i to the market and receives earnings y_i, and the pair $x_i = (y_i, -z_i)$ denotes i's *transaction*. Negative numbers denote a transaction from a spouse to the market, while positive numbers denote the opposite. We denote the household's transactions by $\boldsymbol{x} = (x_f, x_m) = (y_f, -z_f, y_m, -z_m)$. Each spouse's transaction is restricted by their endowment, $y_i \geq 0$ and $0 \leq z_i/w_i \leq 1$.

Additionally, household transactions can be restricted by taxes (or more general institutional environments). To formalize these restrictions, we assume that the spouses choose messages or reports, $\boldsymbol{s} = (s_f, s_m) \in S \times S$, which are mapped into transactions by $\mathbf{x}\colon S \times S \mapsto X \times X$. This restricts transactions to be *decentralized* in the sense that a household's transactions do not depend on other households' messages. The distinction between messages and transactions will be useful when we formalize the mechanism design problem.[5]

The set of all feasible transactions is, therefore,

$$\mathscr{X} = \{\boldsymbol{x} \in X \times X \mid \exists (s_f, s_m) \in S \times S \text{ such that } \boldsymbol{x} = \mathbf{x}(s_f, s_m)\}.$$

Finally, given $\boldsymbol{x} = (y_f, -z_f, y_m, -z_m)$, the (conditional on $\boldsymbol{x}$) utility possibility set for a $\boldsymbol{w}$-household is

$$\mathbf{U}(\boldsymbol{x} \mid \boldsymbol{w}) := \Big\{ (u_f, u_m) \,\Big|\, (u_f, u_m) = (v(c_f, l_f), v(c_m, l_m)) \text{ with } c_f + c_m \leq y_f + y_m, \quad l_f = 1 - \frac{z_f}{w_f}, \quad l_m = 1 - \frac{z_m}{w_m} \Big\},$$

[4] In Section 4, we allow for public goods and household production, but we abstract from these considerations in this section for simplicity.

[5] A tax schedule is framed in this formulation by taking $S = X$, which equates a message to a suggested transaction for each spouse. The map $\mathbf{x}$ then can be defined as $\mathbf{x}(\boldsymbol{x}) = \boldsymbol{x}$ if the transactions are feasible according to the tax schedule and zero otherwise. For more details, see Section 3.3. Independence with respect to other households' reports characterizes a tax schedule and is compatible with feasibility for any mechanism under the assumption of assignment uncertainty as explained by Roberts (1984).

2.2 The Household Decision Process

Spouses have two decisions to make in our model. They decide their message s_i, which generates the household's transactions via the function $\mathbf{x}(s_f, s_m)$, and decide how to split their earnings $y_f + y_m$ into individual consumptions, c_f and c_m.

The decision-making process of a household depends on whether spouses are in agreement or disagreement. When in agreement, the spouses Nash-bargain to decide their messages and allocations, which are the decisions observed in equilibrium. On the other hand, when in disagreement, each spouse chooses messages non-cooperatively to maximize his/her utility. The utilities that arise as the non-cooperative outcome of these decisions define the threat points of the Nash-bargaining used in the agreement state.

Agreement State Given disagreement utilities $\bar{\boldsymbol{u}}(\boldsymbol{w}) = (\bar{u}_f(\boldsymbol{w}), \bar{u}_m(\boldsymbol{w}))$, which will be defined in (3), a $\boldsymbol{w}$-household chooses messages (s_f, s_m) cooperatively to maximize the Nash product, $\big(u_f - \bar{u}_f(\boldsymbol{w})\big)\big(u_m - \bar{u}_m(\boldsymbol{w})\big)$, i.e., the household solves

$$\max_{(s_f,s_m)\in S_f\times S_m} \left\{ \max_{(u_f,u_m)\in\mathbf{U}(\mathbf{x}(s_f,s_m)|\boldsymbol{w})} \Big(u_f - \bar{u}_f(\boldsymbol{w})\Big)\Big(u_m - \bar{u}_m(\boldsymbol{w})\Big) \right\}. \tag{1}$$

The solution (s_f^*, s_m^*) defines the household's transactions and utilities in equilibrium,

$$\boldsymbol{x}(\boldsymbol{w}) = \mathbf{x}(s_f^*, s_m^*) \quad \text{and} \quad \boldsymbol{u}(\boldsymbol{w}) = \underset{(u_f,u_m)\in\mathbf{U}(\mathbf{x}(s_f^*,s_m^*)|\boldsymbol{w})}{\operatorname{argmax}} (u_f - \bar{u}_f(\boldsymbol{w}))(u_m - \bar{u}^m(\boldsymbol{w})).$$

Disagreement State For the disagreement state, we assume there is a function $\bar{\mathbf{u}}(\,\cdot \mid \boldsymbol{w}) : X \times X \mapsto U \times U$ that maps transactions into a pair of disagreement utilities such that $\bar{\mathbf{u}}(\boldsymbol{x} \mid \boldsymbol{w}) \in \mathbf{U}(\boldsymbol{x} \mid \boldsymbol{w})$. We abstract from the details leading to this function and, instead, only impose that the function $\bar{\mathbf{u}}(\,\cdot \mid \boldsymbol{w})$ does not depend on the message space, S, or the function $\mathbf{x}$ mapping messages to transactions. In other words, disagreement utilities only depend on disagreement transactions and are *independent* of the institutional environment (or mechanism) leading to those transactions.[6]

Given $\bar{\mathbf{u}}(\cdot \mid \boldsymbol{w}) = (\bar{\mathrm{u}}_f(\cdot \mid \boldsymbol{w}), \bar{\mathrm{u}}_m(\cdot \mid \boldsymbol{w}))$, each spouse decides their message to the planner non-cooperatively to maximize their disagreement utility $\bar{\mathrm{u}}_i(\cdot \mid \boldsymbol{w})$. That is, the wife chooses s_f to

$$\max_{s_f\in S} \bar{\mathrm{u}}_f(\mathrm{x}_f(s_f, s_m) \mid \boldsymbol{w}), \tag{2}$$

[6]While our theoretical results do not depend on specifying how these rules are determined, these functions play a crucial role in the practical implementation of proposed policies. We will, therefore, impose more structure for the quantitative exercises. Importantly, in Section 4, we endogenize the function $\bar{\mathbf{u}}$, by assuming that, beyond social norms, conditions in the marriage market affect this function through commitments that allow for expected utility transfers. This allows us to formalize the general equilibrium notion discussed by Lundberg and Pollak (1993).

taking her husband's choice, s_m, as given. Similarly, the husband chooses s_m to maximize his disagreement utility taking s_f as given. Let $(\hat{s}_f, \hat{s}_m)$ be a (pure-strategy) Nash equilibrium of this game, then threat points are defined as

$$\bar{\boldsymbol{u}}(\boldsymbol{w}) = \left(\bar{u}^f(\boldsymbol{w}), \bar{u}^m(\boldsymbol{w})\right) := \bar{\mathbf{u}}\left(\mathrm{x}^f(\hat{s}_f, \hat{s}_m), \mathrm{x}^m(\hat{s}_m, \hat{s}_f); \boldsymbol{w}\right). \tag{3}$$

Remark. Our focus on pure-strategy equilibria is for notational convenience only since we could define the threat point as the expected value over the mixed-strategy equilibrium. Similarly, if there are multiple Nash equilibria, we assume the spouses agree on the frequency of each equilibrium and take the expected disagreement utility over this frequency as the threat point.

3 Mechanisms Design

Proofs for this section are in Appendix A.
The message set, S, and the mapping from messages to transactions, $\mathbf{x}$, define a mechanism.[7]

Definition. A *mechanism*, $\mathcal{M}$, consists of a message space, $S_{\mathcal{M}}$, and an outcome function, $\mathbf{x}_{\mathcal{M}} : S_{\mathcal{M}} \times S_{\mathcal{M}} \mapsto X \times X$ mapping messages from both spouses into transactions.[8] The space of possible transactions induced by $\mathcal{M}$ is denoted by $\mathcal{X}_{\mathcal{M}} = \mathbf{x}_{\mathcal{M}}\left(S_{\mathcal{M}} \times S_{\mathcal{M}}\right)$, which is a subset of the set of all possible transactions, $\mathcal{X}$.

In all that follows, we index message sets, utility possibility sets, disagreement utilities, etc. by the mechanism's name.

Definition. *The allocation implemented by* $\mathcal{M}$, $\mathbf{A}_{\mathcal{M}}$, is the set of transactions and utilities for every household,

$$\mathbf{A}_{\mathcal{M}} := \left\{\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}), \boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w})\right\}_{\boldsymbol{w}},$$

that arise through the decision process described by equations (1) and (2).

Remark. Many disagreement utilities can implement the same transactions and agreement utilities. Since the disagreement state is off the equilibrium path, it would be too stringent to add disagreement transactions and utilities as part of the definition of an allocation.

A particularly important example of a mechanism is a tax schedule, which we formally define in Section 3.3.

[7] Also referred to as a game form in the literature, e.g. Roberts (1984); Guesnerie (1998).

[8] It is without loss of generality to assume that the message space for each spouse is the same since if we have S_f and S_m, we can take $S = S_f \cup S_m$ and assume that $\mathbf{x}(s_f, s_m) = \mathbf{0}$ whenever $s_i \in S \setminus S_i$.

3.1 Revelation Principle

In our definition of a mechanism, there is the implicit constraint that the planner does not know *a priori* the household members' productivities nor if spouses are in agreement or disagreement; only the household members themselves know it. For this reason, productivities and states must be elicited by the mechanism.

In this section, we show that this information defines the spouses' type and that a *revelation principle* holds: any allocation implemented by a general mechanism can be implemented by a *direct mechanism* asking productivities and states from each spouse.

Definition. In a $\boldsymbol{w}$-household, spouse i type, τ_i is a triplet including their productivity, their partner's productivity, and the household state. That is

$$\tau_i = (w_f, w_m, t), \qquad t \in \{a, d\}$$

where $t = a$ if the household is in agreement or $t = d$ if they are in disagreement. Therefore, in a household, spouses share the same type.

Definition. A *direct mechanism*, $\mathcal{D}$, is a mechanism for which the message space is the set of possible types, $S_{\mathcal{D}} := W \times W \times \{a, d\}$.

The next result extends the classical revelation principle—Myerson (1979); Dasgupta, Hammond, and Maskin (1979); Harris and Townsend (1981)—to our multi-agent setting.

Proposition 1 (Revelation Principle)**.** *Let* $\mathbf{A}_{\mathcal{M}} = \{\boldsymbol{x}_{\mathcal{M}}, \boldsymbol{u}_{\mathcal{M}}\}$ *be the allocation implemented by* $\mathcal{M}$*. Then there exists a direct mechanism,* $\mathcal{D}$*, such that spouses truthfully report their types, and* $\mathbf{A}_{\mathcal{D}} = \mathbf{A}_{\mathcal{M}}$*.*

The proof is in Appendix A and is standard; nevertheless, this proposition is significant as it shows that our *definition* of type is the correct one. An alternative interpretation of the result is that our definition of type has the minimal information necessary for the revelation principle to hold.

Remark. Since spouses act cooperatively in agreement, implementable allocations must be incentive-compatible with the household's objective. Therefore, even though spouses in a household share the same type, the planner cannot explore this correlation to implement any (feasible) allocation as in Crémer and McLean (1988).

Remark. The disagreement game may have more Nash equilibria under the direct mechanism than under the original mechanism. This is a standard issue in mechanism implementation and, following the literature, we assume that the Nash equilibrium that would be played in disagreement (or the frequency of each Nash-equilibria) is the same as in the original mechanism.

3.2 Incentive-feasible Allocations

There are two restrictions to implementable allocations. The first (and most relevant for our purposes) is the incentive-compatibility restriction. The other restriction is the technological feasibility, which we represent with a function G such that an allocation $\mathbf{A}_{\mathcal{M}} := \{\boldsymbol{x}_{\mathcal{M}}, \boldsymbol{u}_{\mathcal{M}}\}$ is *feasible* if and only if

$$G\left(\sum_{\boldsymbol{w}} \sum_{i=f,m} x_{i,\mathcal{M}}(\boldsymbol{w})\mu(\boldsymbol{w})\right) \leq 0,$$

where $\mu(\boldsymbol{w})$ is the proportion of household with productivity $\boldsymbol{w}$.

An allocation from the direct mechanism $\mathcal{D}$, $\mathbf{A}_{\mathcal{D}} = \{\boldsymbol{x}_{\mathcal{D}}, \boldsymbol{u}_{\mathcal{D}}\}$, with $\boldsymbol{x}_{\mathcal{D}}(\boldsymbol{w}) = \mathrm{x}_{\mathcal{D}}(\boldsymbol{w}, a, \boldsymbol{w}, a)$ for all $\boldsymbol{w}$, is *incentive-feasible* if:

i) *Incentive Compatibility (in disagreement)* — For every $\boldsymbol{w}$-households, truth-telling is a Nash-equilibrium for the disagreement game: $(\boldsymbol{w}, d) \in \operatorname{argmax}_{\boldsymbol{w}',\, t'} \bar{\mathrm{u}}_f\left(\mathrm{x}_{\mathcal{D}}(\boldsymbol{w}', t', \boldsymbol{w}, d); \boldsymbol{w}\right)$ and $(\boldsymbol{w}, d) \in \operatorname{argmax}_{\boldsymbol{w}',\, t'} \bar{\mathrm{u}}_m\left(\mathrm{x}_{\mathcal{D}}(\boldsymbol{w}, d, \boldsymbol{w}', t); \boldsymbol{w}\right)$.

ii) *Incentive Compatibility (in agreement)* — Given the disagreement utilities, $\boldsymbol{u}_{\mathcal{D}}(\boldsymbol{w})$, for every $\boldsymbol{w}$-households truth-telling is optimal in the agreement state:

$$(\boldsymbol{w}, a, \boldsymbol{w}, a) \in \underset{\boldsymbol{w}', t', \boldsymbol{w}'', t''}{\operatorname{argmax}} V_{\mathcal{D}}(\mathrm{x}_{\mathcal{D}}(\boldsymbol{w}', t', \boldsymbol{w}'', t''); \boldsymbol{w}),$$

where

$$V_{\mathcal{D}}(\boldsymbol{x}; \boldsymbol{w}) := \max_{(u_f, u_m) \in U(\boldsymbol{x}|\boldsymbol{w})} \left(u_f - \bar{u}_{f,\mathcal{D}}(\boldsymbol{w})\right)\left(u_m - \bar{u}_{m,\mathcal{D}}(\boldsymbol{w})\right) \tag{4}$$

is the $\boldsymbol{w}$-household Nash-bargaining objective function over transactions instead of utilities.

iii) *Feasibility* — $G\left(\sum_{\boldsymbol{w}} \sum_{i=f,m} x_{i,\mathcal{D}}(\boldsymbol{w})\mu(\boldsymbol{w})\right) \leq 0$.

Remark. In (iii), feasibility is only required in agreement because the disagreement allocation does not arise in equilibrium.

Remark. Nowhere in the definition or the proofs did we specify the dimensionality of x or the technology mapping transactions to utility. It is also straightforward to accommodate a richer type-space beyond the productivity w. The model is, therefore, general enough to accommodate greater heterogeneity across households as well as public goods and home production. In Section 5, we rely on this level of generality to incorporate these features in a fully specified household model.

Finally, another characterization of (incentive-feasible) allocations is given below

Proposition 2 (Implementable Allocations)**.** *With differentiable utility functions, an allocation $\mathbf{A}_{\mathcal{M}}$ can be implemented by another mechanism $\mathcal{N}$ if the equilibrium transactions are the same, $\boldsymbol{x}_{\mathcal{N}}(\cdot) = \boldsymbol{x}_{\mathcal{M}}(\cdot)$, and, for every household $\boldsymbol{w}$, their disagreement utilities under $\mathcal{N}$ satisfy*

$$\bar{\boldsymbol{u}}_{\mathcal{N}}(\boldsymbol{w}) = p\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w}) + (1-p)\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{w})$$

for some $p \leq 1$. If $\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{w}) \neq \boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w})$, then this condition is also necessary.

With a characterization of all incentive-feasible allocations, it is natural to ask which allocations can be implemented with institutional environments simpler than a direct mechanism, e.g., tax systems.

3.3 Tax schedules as Mechanisms

A *tax schedule*, T, maps labor supply decisions, $\boldsymbol{z}$, to feasible after-tax earnings, $\boldsymbol{y}$. We say that a transaction is in the budget set induced by the tax schedule T if

$$\boldsymbol{z} - \boldsymbol{y} := (z_f - y_f, z_m - y_m) \in T(\boldsymbol{z}). \tag{5}$$

To formally represent a tax schedule as a mechanism $\mathcal{T}$, take the messages as desired transactions, $\hat{x}_f$ and $\hat{x}_m$, so that $S_{\mathcal{T}} = X$. If these transactions respect the budget set in (5), then the planner allows the transaction, and the outcome function is

$$\mathbf{x}_{\mathcal{T}}(\hat{x}_f, \hat{x}_m) = (\hat{x}_f, \hat{x}_m) .$$

Otherwise, if the desired transactions are outside the budget set, then the planner denies the transaction and sets $\mathbf{x}_{\mathcal{T}}(\hat{x}_f, \hat{x}_m) = \mathbf{0}$.

Most tax schedules do not specify who must remit the taxes from a household or receive tax payments; our definition is more general and allows for targeted payments. However, if a tax schedule is invariant to this consideration, we call it an income-pooling tax schedule.

Definition. A tax schedule T satisfies *income pooling* if for any labor supply $\boldsymbol{z} = (z_f, z_m)$, and earnings $\boldsymbol{y} = (y_f, y_m)$ and $\boldsymbol{y}' = (y'_f, y'_m)$ with $y_f + y_m = y'_f + y'_m$ it holds that

$$\boldsymbol{z} - \boldsymbol{y} = (z_f - y_f, z_m - y_m) \in T(z_f, z_m) \quad \Rightarrow \quad \boldsymbol{z} - \boldsymbol{y}' = (z_f - y'_f, z_m - y'_m) \in T(z_f, z_m).$$

Definition. If a mechanism $\mathcal{T}$ can be induced by a tax schedule T, we call it a *tax-schedule mechanism*. Furthermore, if $\mathcal{T}$ can be induced by an income-pooling tax schedule, then we call it an *income-pooling tax-schedule mechanism*.

Remark. There are two important aspects of tax schedules only relevant to Collective households.

First, which spouse receives the earnings may affect the allocation, so we cannot work only with income-pooling tax schedules; while for unitary households, there is no loss in using only income-pooling tax schedules.

Second, if there are multiple filing options with different tax schedules, say T_1 and T_2, the allocation is not necessarily the same as providing the envelope tax schedule T,

$$T(z_f, z_m) = T_1(z_f, z_m) \cup T_2(z_f, z_m),$$

even if one schedule dominates the other (i.e. $T_2(z_f, z_m) \subset T_1(z_f, z_m)$ for all z_f, z_m); while for unitary households these two scenarios are equivalent.

3.4 Limits to Taxation

For individual agents, the *taxation principle* asserts that tax schedules are as capable as sophisticated mechanisms: any implementable allocation can be implemented with budget-set restrictions.[9]

We show that this result cannot be extended to our Collective model because the household's induced preferences (4) vary with the mechanism (via the disagreement utilities as threat points). If threat points were exogenous, the model would be *unitary* and the taxation principle would hold.

Proposition 3 (Roberts (1984))**.** *A set of transactions $\{\boldsymbol{x}(\boldsymbol{w})\}_{\boldsymbol{w}}$ is implementable by a tax-schedule mechanism $\mathcal{T}$ if and only if it satisfies $V_{\mathcal{T}}(\boldsymbol{x}(\boldsymbol{w}); \boldsymbol{w}) \geq V_{\mathcal{T}}(\boldsymbol{x}(\boldsymbol{w}'); \boldsymbol{w})$ for any $\boldsymbol{w}$ and $\boldsymbol{w}'$.*

An immediate consequence of this proposition is that the taxation principle holds for unitary households since the threat points do not depend on the mechanism.

Corollary 4 (Taxation principle for unitary households)**.** *Suppose threat points are exogenous, in the sense that for any mechanism $\mathcal{M}$ we have $\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{w}) = \bar{\boldsymbol{u}}(\boldsymbol{w})$. Then, for any $\mathcal{M}$, there is an income-pooling tax-schedule mechanism $\mathcal{T}$ that implements the allocation $\mathbf{A}_{\mathcal{M}} = \{\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}), \boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w})\}_{\boldsymbol{w}}$.*

In particular, policies that only change the recipient of government transfers in a household (like the 1979 UK Reform of Child Benefits 'from the pocket to the purse') have no impact if households are unitary.

This explains why the literature that assumes that households are unitary does not specify the details of who pays or receives tax transfers. For unitary households, we can always assume that tax schedules are income pooling. When we move away from unitary models, however, income pooling is too restrictive a constraint even in the simplest of settings.

Corollary 5 (Partial taxation principle for transferable utility)**.** *Suppose the utility function is quasi-linear in consumption, $v(c, l) = c + \hbar(l)$. Then, for any mechanism $\mathcal{M}$, there is an income-pooling*

[9] Guesnerie (1998) refers to a sophisticated (mechanism designer) and an unsophisticated (tax designer) planner, and shows that the set of allocations that one implements are the same as the ones the other implements.

*tax-schedule mechanism $\mathcal{T}$ that implements the set of transactions $\{\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w})\}_{\boldsymbol{w}}$. However, this tax-schedule will **not**, in general, implement the same utilities, $\{\boldsymbol{u}_{\mathcal{T}}(\boldsymbol{w})\}_{\boldsymbol{w}} \neq \{\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w})\}_{\boldsymbol{w}}$.*

Given a tax-schedule $\widetilde{\mathcal{T}}$ that implements the transactions $\{\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w})\}_{\boldsymbol{w}}$, it implements the same utilities as the mechanism $\mathcal{M}$ if and only if, for every $\boldsymbol{w}$,

$$u_{f,\mathcal{M}}(\boldsymbol{w}) - \bar{u}_{f,\widetilde{\mathcal{T}}}(\boldsymbol{w}) = u_{m,\mathcal{M}}(\boldsymbol{w}) - \bar{u}_{m,\widetilde{\mathcal{T}}}(\boldsymbol{w}). \tag{6}$$

For the case of transferable utility, there is some hope of a taxation principle if we allow general, non-income-pooling tax schedules because we can divide the problem into two steps. First, find a tax schedule that implements the same transactions as the mechanism. Then, for the disagreement transactions induced by the tax schedule (which *do not* need to be the same as the ones induced by the mechanism), judiciously split the total earnings $\bar{y}_f + \bar{y}_m$ so that (6) is satisfied.

However, this may not be possible even in the transferable utility case because who receives the earnings is only part of how disagreement utilities are decided. To implement the same allocation as a mechanism, the planner may need to provide contradictory labor supply incentives in the agreement and disagreement states. The next (extreme) example makes the point clearer as the planner wants husbands to not work in disagreement.

Example 3.1 (Taxation principle failure)**.** *Consider an economy where women have zero productivity, $w_f = 0$, and there is a continuum of men with productivity uniformly distributed in $[0, 1]$. Each agent has utility $v(c, \ell) = c + \log(\ell)$, and the planner has a preference for equality with welfare function*[10]

$$W(\{\boldsymbol{u}(w_m)\}_{w_m}) = \int_0^1 \log(u_f(w)) + \log(u_m(w)) \, \mathrm{d}w$$

In disagreement, consumption is split according to

$$\bar{c}_i = \frac{\bar{z}_i}{\bar{z}_f + \bar{z}_m}(\bar{y}_f + \bar{y}_m) = \begin{cases} 0 & \text{if} \quad i = f \\ \bar{z}_m - T(\bar{z}_m) & \text{if} \quad i = m \end{cases}.$$

It follows that $\bar{u}_f = 0$.

In agreement, given threat points $\bar{\boldsymbol{u}} = (0, \bar{u}_m)$ and a tax schedule $T(z)$, the household in agreement solves the Nash-bargaining objective,

$$\max_{c_f, c_m, z_m} c_f \cdot \left[c_m + \log\left(1 - \frac{z_m}{w_m}\right) - \bar{u}_m\right],$$

subject to $c_f + c_m = z_m - T(z_m)$. For this specification, the optimal z_m does not depend on $\bar{u}_m$

[10] Since only men vary in productivity, we index households by w_m instead of $\boldsymbol{w} = (0, w_m)$.

and $u_f(w_m) = u_m(w_m) - \bar{u}_m$.[11] *Since the planner dislikes inequality, the optimal threat point has $\bar{u}_m = 0$.*

However, to implement $\bar{u}_m = 0$, the planner would need to heavily tax labor supply, which would not be optimal. A tax schedule is not enough to both implement the optimal threat points and the optimal transactions conditional on threat points. *And, in fact, **any** tax-schedule mechanism implements allocations with*

$$u_f(w_m) = \bar{u}_f(w_m) = 0,$$

and welfare is minus infinity in this example. Notice that, since labor supply is independent of threat points, the observed transactions, $(y_m, -z_m)$, are the same no matter the disagreement utilities, but the allocation of (agreement) utilities is vastly different.

This example suggests a potential role for the use of two separate schedules, one for the disagreement game and another one for choices in agreement.

Filing Options If we add to T a second schedule $\overline{T}$ as a *filing option* we get a *two-schedule* tax system. Then, we can map the tax system to a *two-schedule tax mechanism* with a choice set $S_i := X \times \{a, d\}$. If both spouses report a, then they use T; but if one of them reports d, then they must use $\overline{T}$.

With endogenous threat points, a tax-schedule mechanism may not suffice, and a two-schedule tax mechanism can be used to target threat points more precisely. Intuitively, the planner has two instruments for two objectives: T to induce choices conditional on threat points, and $\overline{T}$ to induce better threat points. The proposition below shows that if no spouses have the same earnings in disagreement, a two-schedule tax system is sufficient to implement any (incentive-feasible) allocation.[12]

Proposition 6. *Given a mechanism $\mathcal{M}$, suppose that the disagreement labor earnings, $\bar{z}_{f,\mathcal{M}}(\cdot)$ and $\bar{z}_{m,\mathcal{M}}(\cdot)$, are both one-to-one, that is we can recover $\mathbf{w}$ from each spouse separately, then there is a two-schedule tax system that implements $\mathbf{A}_{\mathcal{M}}$.*

However, even a two-schedule tax system may not implement all incentive-feasible allocations. For example, if two wives have low productivity and decide not to work in disagreement, then a tax mechanism cannot costlessly separate their husbands, while a direct mechanism can.

[11] Since women do not work,

$$u_f(w_m) := c_f(w_m) \qquad \text{and} \qquad u_m(w_m) := c_m(w_m) + \log\left(1 - \frac{z_m(w_m)}{w_m}\right)$$

[12] These conditions are only sufficient. A planner could implement an allocation in which spouses from different households had the same earnings if their partners did not envy each other.

3.5 Implementable Threat Points

In the previous discussion, we explored the limitations of a tax system. It is also worth exploring what limits the *direct mechanism* from implementing threat points. According to Propositions 7 and 8 below, the only restriction is that no household in agreement should envy a disagreement transaction; otherwise, the planner is free to act without any constraints because spouses in disagreement behave non-cooperatively.

Proposition 7. *Given a mechanism $\mathcal{M}$ and any possible transaction for this mechanism, $\boldsymbol{x} \in \mathcal{X}_{\mathcal{M}}$, there is a mechanism $\mathcal{N}$ that changes the $\boldsymbol{w}$-household's disagreement utility to $\bar{\mathbf{u}}_{\mathcal{N}}(\boldsymbol{w}) = \mathbf{u}(\boldsymbol{x} \mid \boldsymbol{w})$, without affecting the other households' allocations: for all $\boldsymbol{w}' \neq \boldsymbol{w}$, $\boldsymbol{x}_{\mathcal{N}}(\boldsymbol{w}') = \boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}')$ and $\boldsymbol{u}_{\mathcal{N}}(\boldsymbol{w}') = \boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w}')$.*

The non-cooperative behavior of spouses in disagreement allows for a mechanism to affect threat points at zero cost. The only restriction is that no couple in agreement would, instead, announce to be in disagreement, and Proposition 7 provides a sufficient condition for this to be the case. In contrast, a necessary condition is that disagreement transactions cannot be strictly larger than any agreement transactions.

Proposition 8. *Given a mechanism $\mathcal{M}$, if $\bar{\mathbf{u}}$ can be implemented as disagreement utility for a $\boldsymbol{w}$-household without affecting the other households' allocations, then there is a feasible transaction $\boldsymbol{x} \in \mathcal{X}$ such that $\bar{\mathbf{u}} = \bar{\mathbf{u}}(\boldsymbol{x} \mid \boldsymbol{w})$ and $\boldsymbol{x} \not> \boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}')$ for all $\boldsymbol{w}' \neq \boldsymbol{w}$.*

3.6 Better Allocations

From Section 3.5, we know the planner enjoys much latitude to manipulate threat points. Here, we heuristically discuss two ways in which the planner can use this latitude to produce better allocations.

Threat points and dissonance Assume that utility is transferable between spouses, which implies that the utility frontier is linear and household transactions are independent of threat points.[13] The agreement utilities are determined by the orthogonal projection of the threat point $\bar{\boldsymbol{u}} = (\bar{u}_f, \bar{u}_m)$ into the utility frontier as shown in Figure 1.

A planner who dislikes inequality within households would prefer that utility is evenly split. However, at the original threat points, $\bar{\boldsymbol{u}}$, the Nash-bargaining objective favors the husband. This misalignment between the planner's objective and the household's objective is called *dissonance*.

Suppose the planner, by manipulating the mechanism or tax system, can implement $\bar{\boldsymbol{u}}'$ in Figure 1 without affecting the equilibrium allocation for other households, then the planner has an incentive to do so as this eliminates the dissonance.

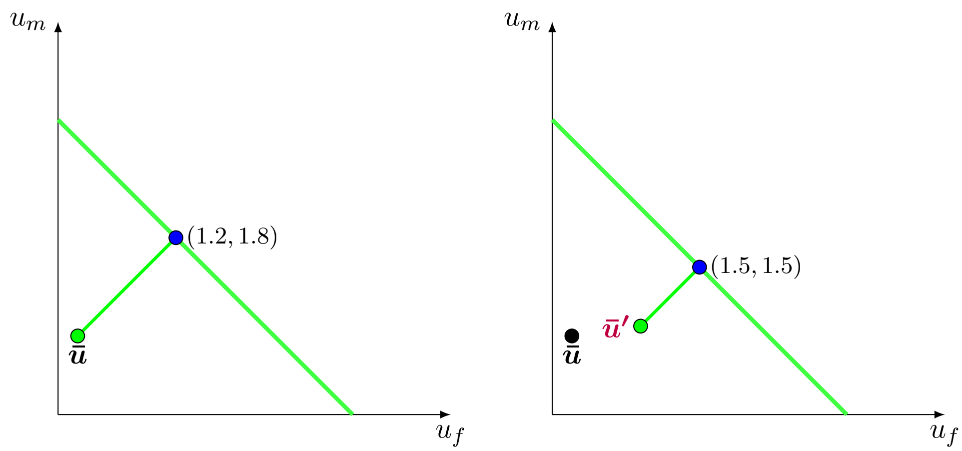

Figure 1: **Aligning Objectives:** The panel in the left displays the set of individually rational utility pairs, when the threat point — green (light) dot — is induced by the tax schedule in place. The blue (dark) dot denotes the optimal choice for the couple. If the planner can induce a different threat point — green dot in the right panel — a new choice distribution of utilities is induced for the same transactions.

Threat points and the ETI. Suppose a household can choose between two possible transactions, $\boldsymbol{x}_g$ and $\boldsymbol{x}_b$ (g for green and b for blue). Figure 2 shows in light green the utility frontier set if the household receives $\boldsymbol{x}_g$, while in dark blue is the frontier if they receive $\boldsymbol{x}_b$. If the threat point is given by $\bar{\boldsymbol{u}}$, then the point A is the agreement utilities with transactions $\boldsymbol{x}_g$, while point B is the agreement utilities with transactions $\boldsymbol{x}_b$ as shown in the left panel of Figure 2.

Assume that the Nash-bargaining objective is the same at A and B for the threat point $\bar{\boldsymbol{u}}$. That is, the household's is "indifferent" between transactions $\boldsymbol{x}_g$ and $\boldsymbol{x}_b$.[14] In this case, the planner cannot increase the taxes for transaction $\boldsymbol{x}_g$ without the household switching to transactions $\boldsymbol{x}_b$. However, if the planner can implement a disagreement utility $\bar{\boldsymbol{u}}'$ in the line connecting $\bar{\boldsymbol{u}}$ and A without affecting other households' transactions, then we can show that A is still the agreement utilities when transactions are $\boldsymbol{x}_g$. Also, the agreement utilities when transactions are $\boldsymbol{x}_b$ change to C and the Nash-bargaining objective for A is now strictly larger than for B. That is, the household strictly "prefers" $\boldsymbol{x}_g$ over $\boldsymbol{x}_b$ for the new threat point, which creates room for a larger tax reform.[15]

Essentially, by moving along the curve, we are reducing the flexibility that a couple has to substitute the utility of one spouse for the other—Figure 3. We are in practice changing the relevant elasticities. This reform is useful whenever the original allocation is distorted due to incentive provision, which we formalize in Lemma 12.

One question that one may have is whether the theoretical findings regarding the taxation principle are quantitatively relevant. To have a sense of how important changes in threat points are, in Section

[13] See Lemma 9 in the appendix.

[14] Recall that axiom 'Preference for symmetry', as defined by Zambrano (2016), substitutes for 'symmetry' to deal with the non-convexity in the utility possibility sets.

[15] See Lemma 10 in the appendix for the details.

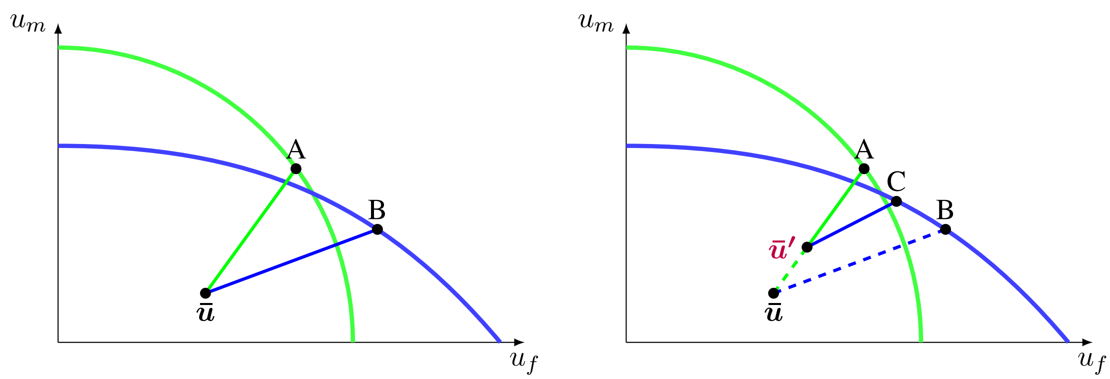

Figure 2: **Breaking Indifference:** The green (light) curve is the frontier of the household's utility possibility for a true report. The blue (dark) curve is the utility frontier for the same couple for the case in which a false report $\boldsymbol{\theta}' \neq \boldsymbol{\theta}$ is made. Point B denotes the utility pair the couple would attain in case of a lie. If the threat points changes from $\bar{\boldsymbol{u}} = (\bar{u}_f, \bar{u}_m)$ to $\bar{\boldsymbol{u}}' = (\bar{u}'_f, \bar{u}'_m)$ equilibrium choices are not changed, yet the deviation utility pair changes from B to C.

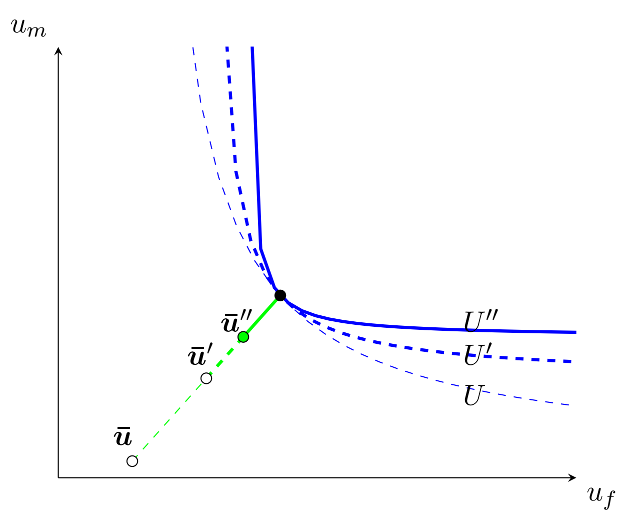

Figure 3: **Changing Elasticities.** The figure shows how the set of utility pairs which are preferred to (u_f^*, u_m^*) shrinks as we move along the curve connecting point a to point (u_f^*, u_m^*).

5 we calibrate a model of the U.S. economy, taking into account the existence of filing options, and evaluate the optimal taxes (and filing options) under different restrictions to the tax system. To take into account the crucial impact of marriage market conditions on the distribution of power between spouses we must first endogenize the household formation.

4 Taking Marriage Markets into Account

In this section, we introduce a marriage market to our analysis, allowing market conditions to affect the distribution of households and spouses' bargaining power. The theoretical results from the previous section survive this extension with minor modifications. Specifically, the Revelation Principle still applies, while the Taxation Principle does not. The characterization of implementable allocations remains essentially unchanged but incorporates the mechanism's effects on both the distributions of households and the disagreement mappings from transactions to utilities. These theoretical results are relegated to Appendix B.

In the marriage market stage, individuals receive a public signal $\tilde{w} \in \widetilde{W} := \{\tilde{w}^1, \ldots, \tilde{w}^T\}$ correlated with their future productivity w, which is unknown at this stage. Following Choo and Siow (2006), agents make marriage decisions anticipating their expected future utility when single or married, which will be a function of their signal $\tilde{w}$ and the prospective partners' signals. The map from transactions to disagreement utilities adjusts to guarantee that the marriage market clears. This summarizes pre-nuptial contracts or any other form of pre-nuptial negotiations that could shift the bargaining power in marriage.

More formally, there is a set $\bar{U}$ of possible disagreement functions $\bar{u}(\cdot|\cdot) : X^2 \times W^2 \to U^2$. This set *does not* depend on the mechanism $\mathcal{M}$.[16] For each possible pair of signals $\tilde{\boldsymbol{w}} = (\tilde{w}_f, \tilde{w}_m)$, there is a disagreement function $\bar{u}^{\tilde{\boldsymbol{w}}}(\cdot|\cdot) \in \bar{U}$. In particular, two couples with the same realized productivities $\boldsymbol{w}$ can have different disagreement utilities if they had different signals during the marriage market stage. Given a mechanism $\mathcal{M}$, we can define for each couple with productivities $\boldsymbol{w}$ and signals $\tilde{\boldsymbol{w}}$ their disagreement utilities $\bar{\boldsymbol{u}}^{\tilde{\boldsymbol{w}}}_{\mathcal{M}}(\boldsymbol{w})$ and agreement utilities $\boldsymbol{u}^{\tilde{\boldsymbol{w}}}_{\mathcal{M}}(\boldsymbol{w})$ as in Section 2.2. At the marriage stage, each prospective spouse knows the expected utilities for each pair, defined as

$$\mathbb{E}u_{i,\mathcal{M}}(\tilde{\boldsymbol{w}}) := \mathbb{E}\left[u^{\tilde{\boldsymbol{w}}}_{i,\mathcal{M}}(\boldsymbol{w}) \mid \tilde{\boldsymbol{w}}\right],$$

where the expectation is over the productivities $\boldsymbol{w}$. They also know their expected utility as singles $\mathbb{E}u_{i,\mathcal{M}}(\tilde{w}_i)$. As in Choo and Siow (2006), there are (private, idiosyncratic) shocks $\boldsymbol{\varepsilon} = (\varepsilon^0, \ldots, \varepsilon^T)$ incorporating non-material aspects of a marriage, so that each woman f with signal $\tilde{w}_f$ chooses their

[16] We claim not that the set is independent of institutions. We are simply not including these institutions among those that can be changed in the analysis. This assumption is the natural generalization of the assumption that the mechanism only affects the disagreement utilities through transactions.

matches based on the utilities

$$\mathbb{E}u_{f,\mathcal{M}}(\tilde{w}_f) + \varepsilon_f^0, \quad \mathbb{E}u_{f,\mathcal{M}}(\tilde{w}_f, \tilde{w}^1) + \varepsilon_f^1, \quad \cdots, \quad \mathbb{E}u_{f,\mathcal{M}}(\tilde{w}_i, \tilde{w}^T) + \varepsilon_f^T.$$

We further assume that ε_f^t are i.i.d. extreme value type 1 with scale σ, centered at $\xi_{\tilde{w}_f,\tilde{w}^t}$, which implies that the average demand from a woman with signal $\tilde{w}_f$ for men with signal $\tilde{w}_m$ is[17]

$$D_{f,\mathcal{M}}(\tilde{w}_f, \tilde{w}_m) := \frac{\exp(\frac{1}{\sigma}(\mathbb{E}u_{f,\mathcal{M}}(\tilde{w}_f, \tilde{w}_m) + \xi_{\tilde{w}_f,\tilde{w}_m}))}{\exp(\frac{1}{\sigma}\mathbb{E}u_{f,\mathcal{M}}(\tilde{w}_f)) + \sum_{t=1}^{T} \exp(\frac{1}{\sigma}(\mathbb{E}u_{f,\mathcal{M}}(\tilde{w}_f, \tilde{w}^t) + \xi_{\tilde{w}_f,\tilde{w}^t}))}.$$

The demand from men, $D_{m,\mathcal{M}}(\tilde{w}_f, \tilde{w}_m)$, follows the same stochastic assumptions.

Marriage Market Equilibrium Given a mechanism $\mathcal{M}$, the marriage market clears if there are disagreement functions $\bar{u}^{\tilde{\boldsymbol{w}}}(\cdot|\cdot) \in \bar{U}$, such that for every signal pair $(\tilde{w}_f, \tilde{w}_m)$,

$$D_{f,\mathcal{M}}(\tilde{w}_f, \tilde{w}_m) = D_{m,\mathcal{M}}(\tilde{w}_f, \tilde{w}_m).$$

This market clearing process is inspired by the model of Gayle and Shephard (2019) where marital bargaining weights adjust to clear the marriage market. However, in their model, the bargaining power is fully determined in the marriage market and there is no room for bargaining in marriage.

Under mild restrictions on the mechanism and the set of feasible disagreement functions, $\bar{U}$, there is a unique equilibrium distribution, $\pi_{\text{MM}}(\cdot)$, that clears the market,

$$\pi_{\text{MM}}(\tilde{w}_f, \tilde{w}_m) = D_{f,\mathcal{M}}(\tilde{w}_f, \tilde{w}_m) = D_{m,\mathcal{M}}(\tilde{w}_f, \tilde{w}_m).$$

This existence and uniqueness result relies heavily on the formalization of matching models with imperfectly transferable utility of Galichon, Kominers, and Weber (2019) and is further discussed in Appendix B.2.

Remark. The restriction that the disagreement function is determined at the marriage stage excludes contracts contingent on productivities. This restriction—reminiscent of incomplete contracts as in Grossman and Hart (1986)—allows for some bargaining to happen *in* marriage; otherwise, agents could contract directly on threat points, and all bargaining would happen *before* marriage.[18]

[17] The vector $\boldsymbol{\xi} = (\xi_{\tilde{w}_f,\tilde{w}_m})$ represents unobserved utility gains from certain matches. For example, if $\xi_{\tilde{w}_f,\tilde{w}_m}$ are higher when $\tilde{w}_f = \tilde{w}_m$, then there is a preference for assortative matches beyond any material aspects. The shock for staying single, ε_0, is centered at 0.

[18] A similar point is raised by Lundberg and Pollak (1993), namely that transfers in the marriage market could undo policies that aim to shift funds from one spouse to another in the marriage. See Pollak (2019) for further discussion on the distinction between *Biding Agreements at the Marriage Market* and *Bargaining-in-Marriage*.

5 A Fully Specified Household Model

In this section, we provide a parametric household model by specifying preferences, technology, and, more importantly, the mapping from transactions to disagreement utilities.

Goods, Preferences, and Technology Individuals have a productivity $w \in W$ and are endowed with 1 unit of time, which can be spent in home production, h, labor supply, z/w, or leisure. Utility depends on consumption, c, public goods, $\hbar_1$ and $\hbar_2$, and leisure, ℓ. It is also affected by the presence of children or not, $b \in \{\mathrm{k}, \mathrm{nk}\}$, and represented by $v^b(c, \hbar_1, \hbar_2, \ell)$,

$$v^b(c, \hbar_1, \hbar_2, \ell) := \log(c) + \beta_1^b \frac{\hbar_1^{1-\gamma_1^b} - 1}{1 - \gamma_1^b} + \beta_2^b \frac{\hbar_2^{1-\gamma_2^b} - 1}{1 - \gamma_2^b} + \beta_\ell \frac{\ell^{1-\gamma_\ell} - 1}{1 - \gamma_\ell}.$$

Households can be a couple or a single agent. Given transactions, $x = (y, -z)$, a single agent with productivity w faces the restrictions

$$c \leq y - \boxed{\mathrm{fc}^b \mathbb{1}[z > 0],} \quad \hbar_1 + \hbar_2 \leq h, \quad \ell \leq 1 - \frac{z}{w} - h,$$

where the highlighted term represents constant fixed costs of working, which are higher if a household has children, $0 < \mathrm{fc}^{\mathrm{nk}} \leq \mathrm{fc}^{\mathrm{k}}$.

For couples, a $\boldsymbol{w}$-household with transactions $\boldsymbol{x} = (y_f, -z_f, y_m, -z_m)$ faces the restrictions

$$c_f + c_m \leq y_f + y_m - \mathrm{FC}_{\boldsymbol{z}}^{k,t}, \quad \boxed{\hbar_1 \leq h_f,\ \hbar_2 \leq h_m,}$$
$$\ell_f \leq 1 - \frac{z_f}{w_f} - h_f, \text{ and } \ell_m \leq 1 - \frac{z_m}{w_m} - h_m. \quad (7)$$

The highlighted terms in Equation (7) incorporate gender specialization; for couples, women contribute to public good $\hbar_1$, while men to $\hbar_2$.[19] Fixed costs $\mathrm{FC}_{\boldsymbol{z}}^{k,t}$ can depend on the couple agreement state, $t = a$ or $t = d$. We assume that couples in disagreement always pay the fixed costs,[20] while couples in agreement pay the fixed costs only if both spouses are working,

$$\mathrm{FC}_{\boldsymbol{z}}^{b,d} = \mathrm{fc}^b, \qquad \mathrm{FC}_{\boldsymbol{z}}^{b,a} = \mathrm{fc}^b \mathbb{1}[z_f > 0, z_m > 0].$$

Labor supply in agreement A couple in agreement has four possible labor supply outcomes in the extensive margin: neither work, $\mathbf{z} = (0,0)$, only one works, $\mathbf{z} \in \{(0,1),(1,0)\}$, or both work,

[19]This assumption is inspired by the 'separate spheres' concept of Lundberg and Pollak (1993), which provides a formalization to socially enforced gender roles within the marriage. Public goods are also a natural way to incorporate gains from marriage and the potential inefficiencies of a disagreement state.

[20]This assumption guarantees that if the equilibrium selection in disagreement is continuous with respect to parameters, then the disagreement utility will be as well.

$\mathbf{z} = (1, 1)$. To avoid discontinuities in the household behavior as we calibrate the parameters, we assume that couples decide between which extensive margin happens with probability

$$\frac{(u_f^{\mathbf{z}} - \bar{u}_f)(u_m^{\mathbf{z}} - \bar{u}_m)}{\sum_{\mathbf{z}'}(u_f^{\mathbf{z}'} - \bar{u}_f)(u_m^{\mathbf{z}'} - \bar{u}_m)}.$$

where $u_i^{\mathbf{z}}$ is the utility of spouse i when the couple in agreement is constrained to have $\mathbf{z}$ as their extensive labor supply margin, that is $(\mathbb{1}[z_f > 0], \mathbb{1}[z_m > 0]) = \mathbf{z}$. The sum in the denominator is only over $\mathbf{z}$ with a positive Nash product.

Disagreement State Consider a household with productivity $\boldsymbol{w}$, fertility $b \in \{\mathrm{k}, \mathrm{nk}\}$, and marriage-market signals $\tilde{\boldsymbol{w}}$. Given transactions $\boldsymbol{x}$, their disagreement utilities $\bar{\mathbf{u}}^{\tilde{\boldsymbol{w}}}(\boldsymbol{x}|\boldsymbol{w}, b)$ are defined by

$$\bar{\mathrm{u}}_f^{\tilde{\boldsymbol{w}}}(\boldsymbol{x}|\boldsymbol{w}, b) := v^b\left(\bar{c}_f, \bar{h}_1^b, \bar{h}_2^b, 1 - \bar{h}_1^b - \frac{z_f}{w_f}\right) \text{ and } \bar{\mathrm{u}}_m^{\tilde{\boldsymbol{w}}}(\boldsymbol{x}|\boldsymbol{w}, b) := v^b\left(\bar{c}_m, \bar{h}_1^b, \bar{h}_2^b, 1 - \bar{h}_2^b - \frac{z_m}{w_m}\right),$$

where $(\bar{h}_1^b, \bar{h}_2^b)$ solves

$$\beta_1^b\left(\bar{h}_1^b\right)^{-\gamma_1^b} - \beta_\ell\left(1 - \bar{h}_1^b - \frac{z_f}{w_f}\right)^{-\gamma_3} = 0 \text{ and } \beta_2^b\left(\bar{h}_2^b\right)^{-\gamma_2^b} - \beta_\ell\left(1 - \bar{h}_2^b - \frac{z_m}{w_m}\right)^{-\gamma_3} = 0,$$

and

$$\bar{c}_f = (\bar{\alpha}^{\tilde{\boldsymbol{w}}} + \delta(\boldsymbol{x}))(y_f + y_m), \quad \bar{c}_m = (1 - \bar{\alpha}^{\tilde{\boldsymbol{w}}} - \delta(\boldsymbol{x}))(y_f + y_m), \tag{8}$$

The term $\bar{\alpha}^{\tilde{\boldsymbol{w}}}$ is the channel used at the marriage market to contract on a disagreement function to clear the marriage market. The term $\delta(\boldsymbol{x})$ allows for the redistribution of earnings to affect the consumption share.[21]

Fertility and Productivity Transitions After agents exit the marriage market either married or single, there is an exogenous fertility shock; singles with signal $\tilde{w}$ have children with probability $\pi_{\mathrm{k}}(\tilde{w})$, and couples with probability $\pi_{\mathrm{k}}(\tilde{w}_f, \tilde{w}_m)$. After fertility transitions, each agent draws their productivity w_i with probability $\pi_w(w_i|\tilde{w})$, independent of the fertility draw and marital status.

5.1 Calibration

We calibrate our model to approximate the data and empirical results from Gayle and Shephard (2019) (GS from now on) at our baseline specification. The baseline tax system approximates the U.S. tax

[21]How much latitude the planner has to move consumption across spouses is an empirical question with currently little data. In Section 5.2, we discuss how to potentially calibrate this function.

system in 2006 using the following functional forms for after-tax income,

$$\text{ATI}_*(z) = \lambda_* z^{1-\rho_*}, \qquad \text{for } * \in \{\text{s}, \text{hh}, \text{ms}\},$$
$$\text{ATI}_{\text{mj}}(z_f, z_m) = \lambda_{\text{mj}}(z_f + z_m)^{1-\rho_{\text{mj}}}.$$

where s is the system used for singles without children, hh (head-of-household) is used for singles with children, ms (married filing separately) is for couples in disagreement and applied to each z_i separately, and mj (married filing jointly) is for couples in agreement.[22] For our optimal taxation exercises, we use a flexible tensor-product two-dimensional cubic spline specification.

Marriage-market signals $\tilde{w}$ are represented by educational attainment, and discretized in three groups: High school and below, some college, and college and above. Agents' productivities w are drawn from a discretized log-normal distribution conditional on education $\tilde{w}$ and independent of marriage or fertility status. Parameters are taken from GS and the distribution is discretized to 5 earning levels. Fertility transitions are calibrated using the 2006 American Community Survey (ACS). At the baseline, we take the effect of transactions on disagreement consumption share, $\delta(\boldsymbol{x})$, to be identically zero. The utility parameters and fixed cost minimize, with equal weights, the percentage error between the model and data for married couples of

average working and housing time by gender and fertility;
own- and cross-elasticities of hours worked and home time by gender for a marginal change in the tax system.

These statistics are reported in GS and the calibration fit is presented in the top two displays of Table 1. The calibrated utility parameters are in Table 2. The marriage market shock variance, $\sigma = 0.123$, is taken from GS. See Appendix C for more details about the calibration.

Remark. At the baseline, husbands control a larger fraction of consumption in disagreement for most matches. On average, wives control only 33% of the disagreement after-tax income, which is the average of $\bar{\alpha}$ with weights μ.

[22] Under our specifications, it is always optimal for married couples to file jointly at the baseline. In the actual U.S. tax system, there are some exceptional cases where filing separately generates lower taxes.

%		hours worked, $\frac{z}{w}$ model	data	hours home, h model	data
Wife	kid	25.3	25.5	35.8	36.2
	no kid	37.7	35.0	25.1	23.9
Husband	kid	37.4	41.1	21.1	22.4
	no kid	42.6	40.6	17.2	17.3

		hours worked elasticity, $\frac{\%\Delta z/w}{\%\text{ATI}}$ model	GS	hours home elasticity, $\frac{\%\Delta h}{\%\text{ATI}}$ model	GS
Wife	own	0.24	0.54	-0.17	-0.23
	cross	-0.39	-0.32	0.15	0.14
Husband	own	0.17	0.22	-0.10	-0.20
	cross	-0.15	-0.13	0.12	0.10

		$\bar{\alpha}$			ξ			μ		
		Husband			Husband			Husband		
		$\tilde{w}_1$	$\tilde{w}_2$	$\tilde{w}_3$	$\tilde{w}_1$	$\tilde{w}_2$	$\tilde{w}_3$	$\tilde{w}_1$	$\tilde{w}_2$	$\tilde{w}_3$
Wife	$\tilde{w}_1$	0.31	0.22	0.11	-0.46	-0.57	-0.64	0.15	0.04	0.02
	$\tilde{w}_2$	0.40	0.30	0.17	-0.53	-0.46	-0.50	0.07	0.09	0.05
	$\tilde{w}_3$	0.60	0.50	0.34	-0.56	-0.42	-0.26	0.03	0.05	0.17

Table 1: Baseline statistics, hours are in percentage of total available hours. Married couples: Left—Calibrated wife's disagreement consumption shares, $\bar{\alpha}$. Middle—mean of idiosyncratic shock, $\mathbb{E}[\varepsilon] = \boldsymbol{\xi}$. Right—Empirical baseline marriage distribution, μ. All by education attainment: $\tilde{w}_1$ = High school or below, $\tilde{w}_2$ = Some college, $\tilde{w}_3$ = College or above.

β_1^{k}	β_1^{nk}	β_2^{k}	β_2^{nk}	β_ℓ	fc^{k}
0.144	0.162	0.013	0.042	0.224	$\frac{\$12{,}869}{\text{year}}$
γ_1^{k}	γ_1^{nk}	γ_2^{k}	γ_2^{nk}	γ_ℓ	fc^{nk}
1.337	1.01	2.761	1.775	2.148	$\frac{\$482}{\text{year}}$

Table 2: Calibrated parameters.

5.2 Counterfactual Tax Systems

We consider tax experiments to quantitatively evaluate how relevant is the theoretical threat-point channel discussed previously. To make the experiments compatible with the standard tax policies, we assume that taxes can depend on marital status, fertility, and gender but not on education attainment. We highlight the effects through the internal threat-point channel which is unique to our household model.

Targeted cash transfer A common feature of cash transfer policies is to target a specific household member. Suppose the planner transfers a small amount $\mathrm{d}t$ directly to the wife of the couple with kids and lowest productivity. Given transactions in disagreement $\boldsymbol{x} = (y_f, -z_f, y_m, -z_m)$, assume that disagreement consumption satisfies

$$\bar{c}_f = \bar{\alpha}(y_f + y_m) + (\bar{\alpha} + \delta)\mathrm{d}t \qquad \bar{c}_m = (1 - \bar{\alpha})(y_f + y_m) + (1 - \bar{\alpha} - \delta)\mathrm{d}t.$$

The parameter δ measures how much more of the targeted transfer is consumed by the wife relative to the baseline sharing fraction, $\bar{\alpha}$.

Alternatively, if the transfer is not targeted, then disagreement consumption are

$$\bar{c}_f = \bar{\alpha}(y_f + y_m + \mathrm{d}t) \qquad \bar{c}_m = (1 - \bar{\alpha})(y_f + y_m + \mathrm{d}t),$$

which is equivalent to a targeted transfer with $\delta = 0$. Notice that both targeted and not-targeted transfers generate the same budget set. Hence, conditional on disagreement utilities, both are equivalent in agreement.

We consider how much of an untargeted transfer a wife is willing to forgo to receive a targeted transfer. That is, if we denote by WTP the willingness-to-pay, then we are a looking for a marginal transfer $\mathrm{d}t$ such that

$$\left(\frac{\partial u_f}{\partial t} + \frac{\partial u_f}{\partial \bar{u}}\frac{\partial \bar{u}}{\partial t_0}\right)\mathrm{d}t = (1 - \mathrm{WTP})\left(\frac{\partial u_f}{\partial t} + \frac{\partial u_f}{\partial \bar{u}}\frac{\partial \bar{u}}{\partial t_\delta}\right)\mathrm{d}t,$$

where $\frac{\partial \bar{u}}{\partial t_\delta}$ denotes the effect on the disagreement utilities of a marginal transfer for a given δ. For households where the bargaining power is fixed or completely decided at the marriage stage, the willingness to pay for a targeted transfer is always zero.

Table 3 reports how WTP varies with δ in our model.[23] Implicitly, we are holding the marriage market fixed, consequently, $\bar{\alpha}$ is also held fixed. The range of δ is calibrated based on Almås et al. (2018), which provides a randomized experiment of targeted versus non-targeted cash transfers with wives in Macedonia. They find an average willingness to pay of 19% for targeted cash transfers relative

[23]We can show that WTP is proportional to $\frac{\delta}{1+\delta}$.

to a not-targeted cash transfer, which is replicated in our model with $\delta = 7.2\%$.[24]

WTP	0%	6%	12%	19%	24%	30%	36%
δ	0.0%	2.0%	4.2%	7.2%	9.7%	13.2%	17.3%

Table 3: Targeted vs non-targeted cash transfer. Highlighted replicates WTP from Almås et al. (2018)

Eliminate filing options The U.S. tax system allows married couples to choose between two tax schedules, joint or separate, and filing jointly generates larger total after-tax earnings in almost all cases. For unitary household models, where bargaining power does not depend on policy, or if the bargaining power is completely determined in the marriage market, then eliminating the option to file separately should have no effect.

In contrast, for our collective model, removing the option to file separately affects disagreement utilities, which has a small but meaningful impact on the average hours worked, hours home, participation, and elasticities for market working hours and home working hours. The change in the disagreement tax system also changes the equilibrium marriage market allocation, μ, and the equilibrium disagreement consumption sharing rule, $\bar{\alpha}$. Detailed results are presented in Table 4.[25]

Optimal tax schedule Consider a planner with a utilitarian welfare function. We adapt the optimal taxation exercise of Gayle and Shephard (2019) by computing the optimal (joint but gender-neutral) tax schedule taking into account the marriage market impact, but keeping the disagreement tax schedule fixed. This corresponds to only allowing changes in taxes to affect the disagreement state through $\bar{\alpha}^{\tilde{\boldsymbol{w}}}$, as defined in (8).

We consider the impact on welfare of two common restrictions on the tax schedule: income splitting, $T(z_f, z_m) = T(z_f + z_m)$, and separate taxation of individual labor earnings, $T(z_f, z_m) = T(z_f) + T(z_m)$. Results for these experiments relative to the optimal joint tax schedules are reported in the left panel of Table 5.

We then repeat the exercise, now allowing the planner to affect the sharing rule in disagreement. More precisely, we allow the planner to influence the disagreement consumption,

$$\bar{c}_f = \left(\bar{\alpha}^{\tilde{\boldsymbol{w}}} + \delta(\boldsymbol{w})\right)(y_f + y_m),$$

[24]As highlighted in Almås et al. (2018), "[t]he interpretation of a zero or negative WTP is not straightforward. A woman living in a unitary household would report a zero WTP. A woman with a very high level of bargaining power, so that she is effectively the sole decision-maker in the household, would also report a zero WTP. This would be equivalent to a unitary model where household preferences coincide with the woman's preference. A zero WTP could also be obtained in the case of a woman with no bargaining power, perhaps because of social norms."

[25]Interestingly, even though the effect of removing filing options within a couple always increases the labor supply of one spouse and decreases the other's, the overall effect is that both husbands and wives with children, on average, work less. Conversely, both spouses in couples without children work more, on average.

		hours worked, $\frac{z}{w}$ % change	hours home, h % change
Wife	kid	-2.40	0.97
	no kid	0.74	-0.73
Husband	kid	-1.13	0.50
	no kid	0.35	-0.46

		hours worked elasticity, $\frac{\%\Delta z/w}{\%\text{ATI}}$ % change	hours home elasticity, $\frac{\%\Delta h}{\%\text{ATI}}$ % change
Wife	own	-8.72	-4.50
	cross	-9.73	-10.30
Husband	own	-2.73	-2.94
	cross	-5.15	-2.65

		$\bar{\alpha}$ % change			μ % change		
		Husband			Husband		
		$\tilde{w}_1$	$\tilde{w}_2$	$\tilde{w}_3$	$\tilde{w}_1$	$\tilde{w}_2$	$\tilde{w}_3$
Wife	$\tilde{w}_1$	-0.64	-1.12	5.03	1.75	1.77	1.80
	$\tilde{w}_2$	-0.09	-0.69	1.37	1.74	1.72	2.07
	$\tilde{w}_3$	0.41	0.07	-0.51	1.31	1.40	1.05

Table 4: Eliminate Filing Option. Married couples: Left—Calibrated wife's disagreement consumption shares, $\bar{\alpha}$. Right—Empirical baseline marriage distribution, μ. All by education attainment: $\tilde{w}_1$ = High school or below, $\tilde{w}_2$ = Some college, $\tilde{w}_3$ = College or above.

by choosing the function δ with the constraint that $|\delta(\boldsymbol{w})| \leq 1\%$. This formulation aims to approximate the optimal mechanism allowing the planner to have some freedom to affect the disagreement utilities. However, because the consumption shares $\bar{\alpha}$ readjust to clear the market, some of the distributive effects from δ are undone by the marriage market. The results relative to the optimal joint tax schedule with $\delta(\boldsymbol{w}) = 0$ are presented in the right panel of Table 5.

The joint tax schedule is approximated by a flexible two-dimensional cubic spline with 10 breakpoints in each dimension. See Online Appendix C.2 for more details about the tax schedule specification.

Remark. In our experiment, the ability to influence the disagreement utilities through δ is more important when we restrict the tax schedule to be income splitting than when we restrict it to tax separately. Since income-splitting tax schedules display positive jointness, while separate taxation has zero jointness, our experiment suggests that the evaluation of optimal jointness of a tax schedule depends on the set of instruments available to the planner to influence the bargaining power within a couple.

Optimal Tax Schedule

Tax schedule restriction	Welfare Impact (%)
Income Splitting	-5.4
Separate	-7.8

Optimal Tax Schedule and Sharing Rule

Tax schedule restriction	Welfare Impact (%)
No restriction	2.6
Income Splitting	-1.4
Separate	-6.2

Table 5: Welfare Impact measured as percentual change of government's tax revenue to compensate for the change in welfare relative to the optimal tax schedule with δ identically zero. At baseline, tax revenue corresponds to 14.8% of total pre-tax labor earnings. Left panel restricts $\delta = 0$; right panel restricts $|\delta| \leq 1\%$.

6 Conclusion

This paper offers a framework for using mechanism design in an economy with multi-person households. We model household decision-making in such a way as to account for the empirical evidence – Almås et al. (2018); Armand et al. (2020) – that rationalizes most current income support programs: handing resources to the wife increases her influence in household choices.

Under a suitable definition of household types, the revelation principle applies. The taxation principle, in contrast, is no longer valid. The latter finding is consequential for the use of tax filing options. In our quantitative exercises, we find quantitatively relevant effects of removing these options on hours worked at the market and home, related elasticities, the distribution of households, and spouses' command of resources.

Our approach relies on Lundberg and Pollak's (1993) *internal* threat point idea while formalizing its 'general equilibrium' version to nest all current static approaches for the determination of power within the household. It also has predictions regarding aspects of policy, like the role of in-kind distribution, that escape other approaches.

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A Section 3 proofs

Proposition (Revelation Principle)**.** *Let $\mathbf{A}_{\mathcal{M}} = \{\boldsymbol{x}_{\mathcal{M}}, \boldsymbol{u}_{\mathcal{M}}\}$ be the allocation implemented by $\mathcal{M}$. Then there exists a direct mechanism, $\mathcal{D}$, such that spouses truthfully report their types, and $\mathbf{A}_{\mathcal{D}} = \mathbf{A}_{\mathcal{M}}$.*

Proof of Proposition 1. Given a mechanism, $\mathcal{M} = \{S_i, \boldsymbol{x}\}$, and its allocation, $\mathbf{A}_{\mathcal{M}} = \{\boldsymbol{x}_{\mathcal{M}}, \boldsymbol{u}_{\mathcal{M}}\}$, we divide the proof into two steps. First, we show that if threat points are fixed at $\bar{\boldsymbol{u}}_{\mathcal{M}}$, then we can find a direct mechanism that truthfully implements the equilibrium allocation in agreement. Then we show that this direct mechanism implements the threat points truthfully.

Define a direct mechanism $\mathcal{D} = \{S'_i, \boldsymbol{x}'\}$ with $S'_i = \Omega^2 \times \{a, d\}$ and

$$\boldsymbol{x}'(\boldsymbol{w}, t, \boldsymbol{w}', t') = \begin{cases} \boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}) & \text{if } \boldsymbol{w} = \boldsymbol{w}' \text{ and } t = t' = a \\ \bar{\boldsymbol{x}}_{\mathcal{M}}(\boldsymbol{w}) & \text{if } \boldsymbol{w} = \boldsymbol{w}' \text{ and } t = t' = d \\ \mathbf{0} & \text{otherwise} \end{cases}.$$

Direct truthful mechanism given $\bar{\boldsymbol{u}}_{\mathcal{M}}$. For any $\boldsymbol{w}$, and report $(\boldsymbol{w}', t', \boldsymbol{w}'', t'')$, if threat points are given by $\bar{\boldsymbol{u}}_{\mathcal{M}}$, define

$$V(\boldsymbol{x} \mid \boldsymbol{w}) = \max_{(u_f, u_m) \in \mathbf{U}(\boldsymbol{x}|\boldsymbol{w})} \Big(u_f - \bar{u}_{f,\mathcal{M}}(\boldsymbol{w})\Big)\Big(u_m - \bar{u}_{m,\mathcal{M}}(\boldsymbol{w})\Big).$$

Then, we have

$$
\begin{aligned}
V(\boldsymbol{x}'(\boldsymbol{w}, a, \boldsymbol{w}, a) \mid \boldsymbol{w}) = V(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}); \boldsymbol{w}) &\overset{[1]}{\geq} \\
\max \Big\{ V(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}') \mid \boldsymbol{w}), V(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}'') \mid \boldsymbol{w}), V(\bar{\boldsymbol{x}}_{\mathcal{M}}(\boldsymbol{w}') \mid \boldsymbol{w}), V(\bar{\boldsymbol{x}}_{\mathcal{M}}(\boldsymbol{w}'') \mid \boldsymbol{w}), V(\mathbf{0} \mid \boldsymbol{w}) \Big\} \\
\geq U(\boldsymbol{x}'(\boldsymbol{w}', t', \boldsymbol{w}'', t''); \boldsymbol{w}),
\end{aligned}
$$

which implies that telling the truth is an equilibrium of the direct mechanism $\mathcal{D}$ and, by construction, this equilibrium implements the same allocation as $\mathcal{M}$. In [1] we use that $\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w})$ maximizes the Nash-product V for household $\boldsymbol{w}$.

Direct truthful mechanism implements $\bar{\boldsymbol{u}}_{\mathcal{M}}$. Consider a couple in disagreement with productivities $\boldsymbol{w}$, to show that $\bar{\boldsymbol{u}}_{\mathcal{M}}$ is a Nash equilibrium of the disagreement game assume that one of the spouses, say the wife, announces $(\boldsymbol{w}, d)$. Any different announcement from the husband will result in a zero allocation, so it is optimal to announce $(\boldsymbol{w}, d)$ as well. □

Proposition (Implementable Allocations)**.** *With differentiable utility functions, an allocation $\mathbf{A}_{\mathcal{M}}$ can be implemented by another mechanism $\mathcal{N}$ if the equilibrium transactions are the same, $\boldsymbol{x}_{\mathcal{N}}(\cdot) = \boldsymbol{x}_{\mathcal{M}}(\cdot)$, and, for every household $\mathbf{w}$, their disagreement utilities under $\mathcal{N}$ satisfy*

$$\bar{\boldsymbol{u}}_{\mathcal{N}}(\boldsymbol{w}) = p\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w}) + (1-p)\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{w})$$

for some $p \leq 1$. If $\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{w}) \neq \boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w})$, then this condition is also necessary.

Proof of Proposition 2. Given a couple $\boldsymbol{w}$ and a mechanism $\mathcal{M}$, suppose another mechanism $\mathcal{N}$ implements the same transactions as $\mathcal{M}$ and induces disagreement utilities in the (half)line defined by $\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w})$ and $\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{w})$. Then, by Lemma 11, the utility division for $\mathcal{N}$ and $\mathcal{M}$ are the same, that is $\boldsymbol{u}_{\mathcal{N}}(\boldsymbol{w}) = \boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w})$.

In the other direction, if transactions differ, then $\mathcal{N}$ and $\mathcal{M}$ have different allocations by definition. So, assume they have the same transactions, $\boldsymbol{x}_{\mathcal{N}}(\cdot) = \boldsymbol{x}_{\mathcal{M}}(\cdot)$. Given a couple $\boldsymbol{w}$, the utility frontier of $\mathbf{U}(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}) \mid \boldsymbol{w})$ is smooth if we have differentiable utility functions; therefore, by Lemma 11, the only way $\mathcal{N}$ can implement the same utility split as $\mathcal{M}$ is if disagreement utilities lie in the line defined by $\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w})$ and $\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{w})$. □

Proposition (Roberts (1984))**.** *A set of transactions $\{\boldsymbol{x}(\boldsymbol{w})\}_{\boldsymbol{w}}$ is implementable by a tax-schedule mechanism $\mathcal{T}$ if and only if it satisfies $V_{\mathcal{T}}(\boldsymbol{x}(\boldsymbol{w}); \boldsymbol{w}) \geq V_{\mathcal{T}}(\boldsymbol{x}(\boldsymbol{w}'); \boldsymbol{w})$ for any $\boldsymbol{w}$ and $\boldsymbol{w}'$.*

Proof of Proposition 3. In our notation, Roberts (1984) shows that a set of transactions $\{\boldsymbol{x}(\boldsymbol{w})\}$ is implementable by a tax-schedule mechanism $\mathcal{T}$ if and only if it satisfies

1. (Decentralization) The transaction for a household only depend on their type, not on other households' types.

2. (Horizontal equity) Two households with the same type have the same transactions.

3. (Vertical equity) $V_{\mathcal{T}}(\boldsymbol{x}(\boldsymbol{w}); \boldsymbol{w}) \geq V_{\mathcal{T}}(\boldsymbol{x}(\boldsymbol{w}'); \boldsymbol{w})$ for any $\boldsymbol{w}$ and $\boldsymbol{w}'$.

Conditions 1 and 2 hold by construction; therefore, the characterization in the proposition follows. □

Corollary (Taxation principle for unitary households)**.** *Suppose threat points are exogenous, in the sense that for any mechanism $\mathcal{M}$ we have $\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{w}) = \bar{\boldsymbol{u}}(\boldsymbol{w})$. Then, for any $\mathcal{M}$, there is an income-pooling tax-schedule mechanism $\mathcal{T}$ that implements the allocation $\mathbf{A}_{\mathcal{M}} = \{\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}), \boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w})\}_{\boldsymbol{w}}$.*

Proof of Corollary 4. By Proposition 3, a tax-schedule mechanism $\mathcal{T}$ implements the transactions $\{\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w})\}$ if, for any $\boldsymbol{w}$ and $\boldsymbol{w}'$,

$$V_{\mathcal{T}}(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}); \boldsymbol{w}) \geq V_{\mathcal{T}}(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}'); \boldsymbol{w}). \tag{9}$$

Consider the following tax schedule

$$T(\boldsymbol{z}) = \begin{cases} \{\boldsymbol{z}_{\mathcal{M}}(\boldsymbol{w}) - \boldsymbol{y}_{\mathcal{M}}(\boldsymbol{w})\} & \text{if } \boldsymbol{z} = \boldsymbol{z}_{\mathcal{M}}(\boldsymbol{w}) \\ \emptyset & \text{otherwise} \end{cases}$$

By assumption, $\mathcal{T}$ and $\mathcal{M}$ generate the same disagreement utilities $\bar{\boldsymbol{u}}(\boldsymbol{w})$, so $V_{\mathcal{T}}(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}'); \boldsymbol{w}) = V_{\mathcal{M}}(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}'); \boldsymbol{w})$ for any $\boldsymbol{w}$ and $\boldsymbol{w}'$. Also, because $\mathcal{M}$ implements the transactions $\{\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w})\}$ we know that

$$V_{\mathcal{M}}(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}); \boldsymbol{w}) \geq V_{\mathcal{M}}(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}'); \boldsymbol{w}),$$

and we conclude that $\mathcal{T}$ implements the transactions $\{\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w})\}$.

Furthermore, since $\mathcal{M}$ and $\mathcal{T}$ generate the same threat-points and the same transactions, they generate the same utilities in agreement. So, $\mathcal{T}$ implements the same allocation as $\mathcal{M}$. Since the utility set does not depend on who earns the income, we can always take $\mathcal{T}$ to be an income-pooling tax-schedule mechanism. □

Corollary (Partial taxation principle for transferable utility)**.** *Suppose the utility function is quasi-linear in consumption, $v(c, l) = c + \hbar(l)$. Then, for any mechanism $\mathcal{M}$, there is an income-pooling tax-schedule mechanism $\mathcal{T}$ that implements the set of transactions $\{\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w})\}_{\boldsymbol{w}}$. However, this tax-schedule will* ***not****, in general, implement the same utilities, $\{\boldsymbol{u}_{\mathcal{T}}(\boldsymbol{w})\}_{\boldsymbol{w}} \neq \{\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w})\}_{\boldsymbol{w}}$.*

Given a tax-schedule $\widetilde{\mathcal{T}}$ that implements the transactions $\{\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w})\}_{\boldsymbol{w}}$, it implements the same *utilities as the mechanism $\mathcal{M}$ if and only if, for every $\boldsymbol{w}$,*

$$u_{f,\mathcal{M}}(\boldsymbol{w}) - \bar{u}_{f,\widetilde{\mathcal{T}}}(\boldsymbol{w}) = u_{m,\mathcal{M}}(\boldsymbol{w}) - \bar{u}_{m,\widetilde{\mathcal{T}}}(\boldsymbol{w}). \tag{10}$$

Proof of Corollary 5. The quasi-linear specification implies that the optimal transactions in agreement are independent of threat-points. Therefore, by Proposition 3 and the argument in the proof of Corollary 4, there is an income-pooling tax-schedule mechanism $\mathcal{T}$ that implements the transactions $\{\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w})\}_{\boldsymbol{w}}$.

Given transactions $(y_f, -z_f, y_m, -z_m)$, the household in agreement solves

$$\max_{c_f} \left(c_f - \hbar\left(1 - \frac{z_f}{w_f}\right) - \bar{u}_f\right)\left(y_f + y_m - c_f - \hbar\left(1 - \frac{z_f}{w_f}\right) - \bar{u}_f\right).$$

The first-order condition with respect to c_f gives

$$c_f - \hbar\left(1 - \frac{z_f}{w_f}\right) - \bar{u}_f = c_m - \hbar\left(1 - \frac{z_f}{w_f}\right) - \bar{u}_f.$$

So, any tax-schedule mechanism that implements transactions $\{\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w})\}_{\boldsymbol{w}}$, will implement the same utilities if and only if it satisfies

$$u_{f,\mathcal{M}}(\boldsymbol{w}) - \bar{u}_{f,\widetilde{\mathcal{T}}}(\boldsymbol{w}) = u_{m,\mathcal{M}}(\boldsymbol{w}) - \bar{u}_{m,\widetilde{\mathcal{T}}}(\boldsymbol{w}).$$

□

Proposition. *Given a mechanism $\mathcal{M}$, suppose that the disagreement labor earnings, $\bar{z}_{f,\mathcal{M}}(\cdot)$ and $\bar{z}_{m,\mathcal{M}}(\cdot)$, are both one-to-one, that is we can recover $\mathbf{w}$ from each spouse separately, then there is a two-schedule tax system that implements $\mathbf{A}_{\mathcal{M}}$.*

Proof of Proposition 6. First, conditional on threat points, by Corollary 4, we can find a tax-schedule T, such that the tax-schedule mechanism induced by it, $\mathcal{T}$, implements the transactions $\{\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w})\}$.

Second, define a fall-back schedule $\overline{T}$ so that

$$\overline{T}(\boldsymbol{z}) = \begin{cases} \{(\bar{z}_{f,\mathcal{M}}(\boldsymbol{w}) - \bar{y}_{f,\mathcal{M}}(\boldsymbol{w}), \bar{z}_{m,\mathcal{M}}(\boldsymbol{w}) - \bar{y}_{m,\mathcal{M}}(\boldsymbol{w}))\} & \text{if } \boldsymbol{z} = (\bar{z}_{f,\mathcal{M}}(\boldsymbol{w}), \bar{z}_{m,\mathcal{M}}(\boldsymbol{w})) \\ \emptyset & \text{otherwise} \end{cases}$$

Because $\bar{z}_{i,\mathcal{M}}(\cdot)$ are one-to-one, if a spouse from a couple $\boldsymbol{w}$ uses the fall-back schedule $\overline{T}$ and chooses earnings $\bar{z}_{i,\mathcal{M}}(\boldsymbol{w})$, then the only available transaction for the couple is $(\bar{x}_{f,\mathcal{M}}(\boldsymbol{w}), \bar{x}_{m,\mathcal{M}}(\boldsymbol{w}))$. Therefore $\overline{\mathcal{T}}$ induces the same disagreement utilities as $\mathcal{M}$. Furthermore, because all transaction in $\overline{T}$ were feasible in $\mathcal{M}$, the fall-back schedule does not affect the agreement allocation. □

Proposition. *Given a mechanism $\mathcal{M}$ and any possible transaction for this mechanism, $\bar{\boldsymbol{x}} \in \mathcal{X}_{\mathcal{M}}$, there is a mechanism $\mathcal{N}$ that changes the $\boldsymbol{w}$-household's disagreement utility to $\bar{\boldsymbol{u}}_{\mathcal{N}}(\boldsymbol{w}) = \mathbf{u}(\bar{\boldsymbol{x}} \mid \boldsymbol{w})$, without affecting the other households' allocations: for all $\boldsymbol{w}' \neq \boldsymbol{w}$, $\boldsymbol{x}_{\mathcal{N}}(\boldsymbol{w}') = \boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}')$ and $\boldsymbol{u}_{\mathcal{N}}(\boldsymbol{w}') = \boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w}')$.*

Proof of Proposition 7. Given $\bar{\boldsymbol{x}} \in \mathcal{X}_{\mathcal{M}}$, let $\bar{\boldsymbol{u}} = \bar{\boldsymbol{u}}(\bar{\boldsymbol{x}} \mid \boldsymbol{w})$. The proof of proposition 1 shows that we can construct a direct mechanism $\mathcal{N} = \{\Theta^2 \times \{a, d\}, \boldsymbol{x}\}$ such that $\bar{s}^*_{f,\mathcal{N}}(\boldsymbol{w}) = \bar{s}^*_{m,\mathcal{N}}(\boldsymbol{w}) = (\boldsymbol{w}, d)$ as long as the allocation function $\mathbf{x}_{\mathcal{N}}(\cdot)$ punishes lies harshly. In particular, we can implement $\bar{\boldsymbol{x}}$ in disagreement for couple $\boldsymbol{w}$ without affecting the disagreement choices of any other couple.

For a couple $\boldsymbol{w}' \neq \boldsymbol{w}$, if their disagreement utility are the same, and if transaction $\bar{\boldsymbol{x}}$ was feasible at the original mechanism $\mathcal{M}$, that is, if $\bar{\boldsymbol{x}} \in \mathbf{X}_{\mathcal{M}}$, then their optimal allocation is unchanged and we get $\boldsymbol{x}_{\mathcal{N}}(\boldsymbol{w}') = \boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}')$. $\square$

Proposition. *Given a mechanism $\mathcal{M}$, if $\bar{\mathbf{u}}$ can be implemented as disagreement utility for a $\boldsymbol{w}$-household without affecting the other households' allocations, then there is a feasible transaction $\boldsymbol{x} \in \mathcal{X}$ such that $\bar{\mathbf{u}} = \bar{\mathbf{u}}(\boldsymbol{x} \mid \boldsymbol{w})$ and $\boldsymbol{x} \not> \boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}')$ for all $\boldsymbol{w}' \neq \boldsymbol{w}$.*

Proof of Proposition 8. For any $\boldsymbol{w}'$, $V_{\mathcal{M}}(\boldsymbol{x}; \boldsymbol{w}')$ is strictly increasing in $\boldsymbol{x}$.[26] Therefore, a couple $\boldsymbol{w}'$ would change their equilibrium allocation if $\boldsymbol{x} > \boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}')$ was feasible. $\square$

A.1 Lemmata

Lemma 9. *Assume that utilities representing people's preferences are of the form $u_i(c, l) = c + \hbar(l)$ for $\hbar(\cdot)$ strictly increasing and concave. Then, optimal transactions are independent of threat points.*

Proof. For transactions $\boldsymbol{x}$, define $h_i = \hbar(1 - z_i/\theta_i)$, $i = f, m$. Given these transaction the household solves

$$\max_{c_f} \left(c_f + h_f - \bar{u}_f\right)\left(\alpha y - c_f + h_m - \bar{u}_m\right),$$

for $y = y_f + y_m$.

At the optimum, for the maximization problems above, we have

$$c_f + h_f - \bar{u}_f = \alpha y - c_f + h_m - \bar{u}_m,$$

and the value of the program, for a given $\boldsymbol{x}$ is, therefore,

$$\frac{1}{4}\Big[\alpha y + (h_f - \bar{u}_f) + (h_m - \bar{u}_m)\Big]^2.$$

[26]With respect to the component-wise partial order on X^2.

It follows that transaction $\boldsymbol{x}$ is preferred to $\boldsymbol{x}'$ if and only if

$$\Big[\alpha y + (h_f - \bar{u}_f) + (h_m - \bar{u}_m)\Big]^2 \geq \Big[\alpha y' + (h'_f - \bar{u}_f) + (h'_m - \bar{u}_m)\Big]^2,$$

which is equivalent to $\alpha y + h_f + h_m \geq \alpha y' + h'_f + h'_m$ □

Lemma 10. *Given a mechanism $\mathcal{M}$, assume*

$$V_{\mathcal{M}}(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}); \boldsymbol{w}) = V_{\mathcal{M}}(\boldsymbol{x}'; \boldsymbol{w})$$

for some $\boldsymbol{x}'$ such that

$$\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w}) \notin \underset{\boldsymbol{u} \in \mathbf{U}(\boldsymbol{w}|\boldsymbol{x}')}{\operatorname{argmax}} \Big(u_f - \bar{u}_{f,\mathcal{M}}(\boldsymbol{w})\Big)\Big(u_m - \bar{u}_{m,\mathcal{M}}(\boldsymbol{w})\Big).$$

That is, $\boldsymbol{x}'$ gives the same Nash product but generates a different split in utilities.

Assume that there is another mechanism $\mathcal{N}$ that satisfies

$$\bar{\boldsymbol{u}}_{\mathcal{N}}(\boldsymbol{w}) = p\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w}) + (1-p)\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{w})$$

for $p \in (0,1)$. That is, $\mathcal{N}$ moves the disagreement utility of $\mathcal{M}$ closer to its agreement utility.

Then,

$$U_{\mathcal{N}}(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}); \boldsymbol{w}) > U_{\mathcal{N}}(\boldsymbol{x}'; \boldsymbol{w}).$$

Under the disagreement utilities induced by $\mathcal{N}$, the original transaction is strictly preferred to $\boldsymbol{x}'$.

Proof. Without loss, let the spouses' utilities at the original threat point be $(0,0)$. Let (u_f, u_m) be the corresponding agreement solution. We know, in this case, that $u_f u_m \geq u'_f u'_m$ for any feasible utility pair (u'_f, u'_m).

Using that the function $f(x) = (x-p)\left(\frac{1}{x} - p\right)$ is uniquely maximized at $x = 1$ it follows that if $u_f \neq u'_f$, then

$$\left(\frac{u'_f}{u_f} - p\right)\left(\frac{u'_m}{u_m} - p\right) \leq \left(\frac{u'_f}{u_f} - p\right)\left(\frac{u_f}{u'_f} - p\right) < \left(1 - p^2\right)$$

if $u'_f > p u_f$.

Multiply both sides by $u_f u_m$ to get

$$(u'_f - p u_f)(u'_m - p u_m) < (1-p)^2 u_f u_m = (u_f - p u_f)(u_m - p u_m)$$

□

Lemma 11. *Given two mechanisms $\mathcal{M}$ and $\mathcal{N}$ that implement the same transactions, $\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}) = \boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w})$ for all $\boldsymbol{w}$, then $\mathcal{N}$ implements the same utilities as $\mathcal{M}$ if there is $p \leq 1$ such that*

$$\bar{\boldsymbol{u}}_{\mathcal{N}}(\boldsymbol{w}) = p\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w}) + (1-p)\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{w}). \tag{11}$$

and the utility possibility set $\mathbf{U}(\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{w}) \mid \boldsymbol{w})$ is smooth and convex. This condition is also necessary if $\bar{\boldsymbol{u}}_{\mathcal{M}}(\boldsymbol{w}) \neq \boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w})$.

Proof. The proof follows readily from the analysis in the proof of Lemma 10. Alternatively, notice that we can assume without loss of generality that $u := u_{f,\mathcal{M}}(\boldsymbol{w}) = u_{m,\mathcal{M}}(\boldsymbol{w})$ by re-scaling the utilities.

Then, by symmetry and the assumptions on the utility possibility set, the Nash solution is (u, u) if and only if threat points are symmetric. Therefore, threat points must be of the form $(\gamma u, \gamma u)$ with $\gamma \leq 1$ to implement the Nash solution.

Let the threat point for $\mathcal{M}$ (after re-scaling) be given by $(\gamma_{\mathcal{M}} u, \gamma_{\mathcal{M}} u)$ and suppose (11) is valid, then

$$\bar{\boldsymbol{u}}_{\mathcal{N}} = p(u, u) + (1-p)\gamma_{\mathcal{M}}(u, u).$$

Therefore $\bar{\boldsymbol{u}}_{\mathcal{N}}$ is of the form $(\gamma_{\mathcal{N}} u, \gamma_{\mathcal{N}} u)$ for $\gamma_{\mathcal{N}} = p + (1-p)\gamma_{\mathcal{M}} \leq 1$.

Moreover, if $\gamma_{\mathcal{M}} < 1$, then any $\gamma \leq 1$ can be written as $p + (1-p)\gamma_{\mathcal{M}}$ for $p = \frac{\gamma - \gamma_{\mathcal{M}}}{1 - \gamma_{\mathcal{M}}} \leq 1$. □

Lemma 12. *For a given mechanism $\mathcal{M}$, assume that there is another mechanism $\mathcal{N}$ such that it induces the same equilibrium allocation as $\mathcal{M}$, for a subset $\boldsymbol{\Theta}_{>} \subset \Theta^2$ it increases disagreement utilities as in Lemma 10, and, for the rest, it does not affect the disagreement utilities.*

Define the set $\boldsymbol{\Theta}_{<} \subset \Theta^2$ of households such that for $\mathcal{M}$ their allocation is either envied, in the sense of a binding incentive-compatibility constraint, by no one or only envied by households in the set $\boldsymbol{\Theta}_{>}$. Then, one must be true: No transactions for households in $\boldsymbol{\Theta}_{<}$ are distorted, or; the allocation $\boldsymbol{x}_{\mathcal{M}}(\cdot)$ is constrained inefficient.

Proof. By Lemma 10, households in $\boldsymbol{\Theta}_{<}$ are envied by no one for the mechanism $\mathcal{N}$. Therefore, either their allocations are not distorted or they can be improved while respecting incentive compatibility, which implies that $\boldsymbol{x}_{\mathcal{N}}(\cdot) = \boldsymbol{x}_{\mathcal{M}}(\cdot)$ is constrained inefficient. □

B Marriage Market

B.1 Theoretical Extensions

Definition. A *mechanism*, $\mathcal{M}$, consists of, for each ex-ante $\tilde{\boldsymbol{w}}$ couples types, message spaces, $S_{\mathcal{M}}(\tilde{\boldsymbol{w}})$, and outcome functions, $\mathbf{x}_{\tilde{\boldsymbol{w}},\mathcal{M}} : \tilde{W}^2 \times S_{\mathcal{M}}(\tilde{\boldsymbol{w}})^2 \mapsto X \times X$ mapping messages spouses' messages into transactions; message spaces $S_{\mathcal{M}}(\tilde{w}_f)$ and outcome functions, $\mathbf{x}^f_{\tilde{w}_f,\mathcal{M}} : \tilde{W} \times S_{\mathcal{M}}(\tilde{w}_f) \mapsto X$ mapping massages to transactions for each ex-ante type, $\tilde{w}_f$, of single woman, and; message spaces $S_{\mathcal{M}}(\tilde{w}_m)$ and outcome functions, $\mathrm{x}^m_{\tilde{w}_m,\mathcal{M}} : \tilde{W} \times S_{\mathcal{M}}(\tilde{w}_m) \mapsto X$ mapping massages to transactions for each ex-ante type, $\tilde{w}_m$, of single man.

To adapt the results from Section 3 we expand the definition of a multi-person household type to $\tau_i = (\boldsymbol{\theta}, t)$ where $\boldsymbol{\theta} = (w_f, w_m, \tilde{w}_f, \tilde{w}_m)$, $t \in \{a, d\}$.

Definition. A *direct mechanism*, $\mathcal{D}$, is a mechanism for which the message for each ex-ante household type $\tilde{\boldsymbol{w}}$, is the set of possible types, $S_{\mathcal{D}} := W^2 \times \{\tilde{\boldsymbol{w}}\}^2 \times \{a, d\}$, $S_{f,\mathcal{D}} := W \times \{\tilde{w}_f\}$ for each ex-ante type of single woman, $\tilde{w}_f$, and $S_{m,\mathcal{D}} := W \times \{\tilde{w}_m\}$ for each ex-ante type of single woman, $\tilde{w}_m$.

The next Proposition states that a direct mechanism that only uses the information produced by individuals after the marriage market stage, induces the same marriage market equilibrium, which is the content of Proposition 13, below.

Proposition 13 (Revelation Principle)**.** *Let* $\mathbf{A}_{\mathcal{M}} = \left\{\boldsymbol{x}_{\mathcal{M}}, x^f_{\mathcal{M}}, x^m_{\mathcal{M}}, \boldsymbol{u}_{\mathcal{M}}, \mu_{\mathcal{M}}, \mu^f_{\mathcal{M}}, \mu^m_{\mathcal{M}}\right\}$ *be the allocation implemented by* $\mathcal{M}$*. Then, there exists a direct mechanism,* $\mathcal{D}$*, such that spouses truthfully report their types, and* $\mathbf{A}_{\mathcal{D}} = \mathbf{A}_{\mathcal{M}}$*.*

Proof. See Appendix A. □

Proof of Proposition 13. Let $\mathbf{A}_{\mathcal{M}} = \left\{\boldsymbol{x}_{\mathcal{M}}, x^f_{\mathcal{M}}, x^m_{\mathcal{M}}, \boldsymbol{u}_{\mathcal{M}}, \mu_{\mathcal{M}}, \mu^f_{\mathcal{M}}, \mu^m_{\mathcal{M}}\right\}$ be the allocation implemented by a mechanism $\mathcal{M}$. This means that given the choices made at the marriage market stage regarding the mapping $\bar{\mathbf{u}}$, the threat points that arise as equilibria of the disagreement game induce, for every couple, $\boldsymbol{\theta}$, $\boldsymbol{x}_{\mathcal{M}}(\boldsymbol{\theta})$ as the household's transactions and $\boldsymbol{u}_{\mathcal{M}}$ as the utilities attained.

Let us take $\bar{\mathbf{u}}$ and μ as given. Expanding the proof of Proposition 1 to accommodate singles using standard arguments it is immediate that there is a direct mechanism $\mathcal{D}$ that induces the same allocation $\boldsymbol{x}_{\mathcal{D}}(\boldsymbol{\theta}) = \boldsymbol{x}_{\mathcal{M}}(\boldsymbol{\theta})$ and $\boldsymbol{u}_{\mathcal{D}}(\boldsymbol{\theta}) = \boldsymbol{u}_{\mathcal{M}}(\boldsymbol{\theta})$.

Now, consider the marriage market stage. Under the direct mechanism, the same utility transfers are available if the same $\bar{\mathbf{u}}$ are chosen. Because the planner selects the disagreement equilibrium, the set of feasible expected utility pairs allocations is not expanded by the direct mechanism either. Since choice sets at the marriage market stage did not change, the equilibrium under $\mathcal{M}$ remains an equilibrium under $\mathcal{D}$. □

An allocation from the direct mechanism $\mathcal{D}$,

$$\mathbf{A}_{\mathcal{D}} = \left\{\boldsymbol{x}_{\mathcal{D}}, x^f_{\mathcal{D}}, x^m_{\mathcal{D}}, \boldsymbol{u}_{\mathcal{D}}, \mu_{\mathcal{D}}, \mu^f_{\mathcal{D}}, \mu^m_{\mathcal{D}}\right\},$$

with $\boldsymbol{x}_{\mathcal{D}}(\boldsymbol{\theta}) = \mathrm{x}_{\mathcal{D}}(\boldsymbol{\theta}, a, \boldsymbol{\theta}, a)$ for all $\boldsymbol{\theta}$, $x^f(\theta_f) = \mathrm{x}^f_{\mathcal{D}}(\theta_f)$ for all θ_f, and $x^m(\theta_m) = \mathrm{x}^m_{\mathcal{D}}(\theta_m)$ for all θ_m, is incentive feasible if:

i) *Incentive Compatibility Singles* — For all θ_f, $\theta_f \in \operatorname{argmax}_s v(\mathrm{x}^f_{\mathcal{D}}(s), \theta_f)$; For all θ_m, $\theta_m \in \operatorname{argmax}_s v(\mathrm{x}^m_{\mathcal{D}}(s), \theta_m)$

ii) *Incentive Compatibility (in disagreement)* — For every $\boldsymbol{w}$-households, truth-telling is a Nash-equilibrium for the disagreement game:

$$(\boldsymbol{\theta}, d) \in \operatorname*{argmax}_{\boldsymbol{\theta}',\, t'} \bar{\mathrm{u}}_f \left(\mathbf{x}_{\mathcal{D}}(\boldsymbol{\theta}', t', \boldsymbol{\theta}, d); \boldsymbol{\theta} \right)$$

and

$$(\boldsymbol{\theta}, d) \in \operatorname*{argmax}_{\boldsymbol{\theta}',\, t'} \bar{\mathrm{u}}_m \left(\mathbf{x}_{\mathcal{D}}(\boldsymbol{\theta}, d, \boldsymbol{\theta}', t); \boldsymbol{\theta} \right).$$

iii) *Incentive Compatibility (in agreement)* — Given the disagreement utilities, $\boldsymbol{u}_{\mathcal{D}}(\boldsymbol{\theta})$, for every $\boldsymbol{\theta}$-households truth-telling is optimal in the agreement state:

$$(\boldsymbol{\theta}, a, \boldsymbol{\theta}, a) \in \operatorname*{argmax}_{\boldsymbol{\theta}', t', \boldsymbol{\theta}'', t''} V_{\mathcal{D}}(\mathbf{x}_{\mathcal{D}}(\boldsymbol{\theta}', t', \boldsymbol{\theta}'', t''); \boldsymbol{\theta}),$$

where

$$V_{\mathcal{D}}(\boldsymbol{x}; \boldsymbol{\theta}) := \max_{(u_f, u_m) \in U(\boldsymbol{x}; \boldsymbol{w})} \left(u_f - \bar{u}^f_{\mathcal{M}}(\boldsymbol{\theta}) \right) \left(u_m - \bar{u}^m_{\mathcal{M}}(\boldsymbol{\theta}) \right)$$

is the $\boldsymbol{\theta}$-household Nash-bargaining objective function over transactions instead of utilities.

iv) *Feasibility* — $G \left(\sum_{\boldsymbol{\theta}} \sum_{i=f,m} x_{i,\mathcal{D}}(\boldsymbol{\theta}) \mu_{\mathcal{D}}(\boldsymbol{\theta}) + \sum_{i=f,m} \sum_{\theta_i} x_{i,\mathcal{D}}(\theta_i) \mu^i_{\mathcal{D}}(\theta_i) \right) \leq 0.$

v) *Endogeneity of* $\boldsymbol{\mu}$ — For all $\boldsymbol{\theta}$,

$$\mu_{\mathcal{D}}(\boldsymbol{\theta}) = \mu_{\mathcal{D}}(\boldsymbol{w}, \tilde{\boldsymbol{w}}) = \Pr(\boldsymbol{w} | \tilde{\boldsymbol{w}}) \bar{\mu}_{\mathcal{D}}(\tilde{\boldsymbol{w}}),$$

for all θ_f,

$$\mu^f_{\mathcal{D}}(\theta_f) = \mu^f_{\mathcal{D}}(w_f, \tilde{w}_f) = \Pr(w_f | \tilde{w}_f) \bar{\mu}^f_{\mathcal{D}}(\tilde{w}_f),$$

and, for all θ_m,

$$\mu^m_{\mathcal{D}}(\theta_m) = \mu^m_{\mathcal{D}}(w_m, \tilde{w}_m) = \Pr(w_m | \tilde{w}_m) \bar{\mu}^m_{\mathcal{D}}(\tilde{w}_f),$$

where $\bar{\mu}_{\mathcal{D}}(\cdot, \cdot)$, $\bar{\mu}^f_{\mathcal{D}}(\cdot)$, and $\bar{\mu}^m_{\mathcal{D}}(\cdot)$ are an equilibrium of the marriage market.

Remark. In our description of a mechanism, we have allowed marriage market signals, $\tilde{\boldsymbol{w}}$, to be private information. If the planner can observe the signals, which are public for the marriage market participants, then we must restrict the Direct Mechanism message space to force reports regarding $\tilde{\boldsymbol{w}}$ to be truthful.

The definition is very similar to the one in Section 3. The novel issues are the inclusion of singles – item (i) – and its consequence to the resource constraint – item (iv) –, the endogeneity of the distribution of couples – item (v) –, and the redefinition of married agents' types now depend not only on $(\boldsymbol{w}, t)$ but also on $\tilde{\boldsymbol{w}}$ through its potential impact on the mapping from transactions to disagreement utilities, which is the channel through which expected utility is transferred at the marriage market stage.

B.2 Equilibrium Existence

This section outlines sufficient conditions for a marriage market equilibrium in our general model from Section 4. The argument relies on re-framing the marriage market to fit the framework developed by Galichon et al. (2019).

Recall that, at the marriage stage, couples receive a signal $\tilde{\boldsymbol{w}}$, which is informative about their after-marriage productivity, $\boldsymbol{w}$, and must decide on a disagreement function $\bar{\mathbf{u}}(\boldsymbol{x} \mid \boldsymbol{w})$ to be used after marriage in case of disagreement. The set of available disagreement functions is denoted by $\overline{U}$.

Given a mechanism $\mathcal{M}$, the set of possible expected utilities for a couple with signal $\tilde{\boldsymbol{w}}$ is defined as the expected agreement utility for all possible disagreement utility functions, $\bar{\mathbf{u}} \in \bar{U}$,

$$\mathbb{E}\mathcal{U}_{\mathcal{M}}(\tilde{\boldsymbol{w}}) := \left\{ \boldsymbol{v} \in U^2 \mid \text{There is } \bar{\mathbf{u}} \in \overline{U} \text{ such that } \boldsymbol{v} \leq \mathbb{E}\left[\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w} \mid \bar{\mathbf{u}}) \mid \tilde{\boldsymbol{w}}\right] \right\}.[27]$$

Then, following Galichon et al. (2019), we define the *bargaining set* for a couple (f, m) with signal $\tilde{\boldsymbol{w}}$ and idiosyncratic shocks $\boldsymbol{\varepsilon}_f$ and $\boldsymbol{\varepsilon}_m$ as

$$\mathcal{B}_{\mathcal{M}}^{f,m} = \mathbb{E}\mathcal{U}_{\mathcal{M}}(\tilde{\boldsymbol{w}}) \oplus \left\{ \left(\varepsilon_f(\tilde{w}_m), \varepsilon_m(\tilde{w}_f)\right) \right\},$$

where $\oplus$ is the Minkowski sum.

We restate the assumptions from Galichon et al. (2019) for completeness.

Assumption B.1.

1. *Assume that the bargaining set $\mathcal{B}_{\mathcal{M}}^{f,m}$ satisfies*
   - *(a) closed and nonempty;*
   - *(b) lower comprehensive, if $v'_f \leq v_f$, $v'_m \leq v_m$ and $(v_f, v_m) \in \mathcal{B}_{\mathcal{M}}^{f,m}$, then $(v'_f, v'_m) \in \mathcal{B}_{\mathcal{M}}^{f,m}$;*
   - *(c) bounded above, if $v_f^n \overset{n\to\infty}{\to} \infty$, then for any $v_m \geq -\infty$ there is N such that $n \geq N$ implies $(v_f^n, v_m) \notin \mathcal{B}_{\mathcal{M}}^{f,m}$, and similarly for $v_m^n \to \infty$.*
2. *The distributions of idiosyncratic shocks $\boldsymbol{\varepsilon}_f$ and $\boldsymbol{\varepsilon}_m$ are iid with full support in $\mathbb{R}$ and absolutely continuous with respect to the Lebesgue measure.*
3. *Let $\partial\mathbb{E}\mathcal{U}_{\mathcal{M}}(\tilde{\boldsymbol{w}})$ denote the frontier of the feasible expected utility set, then*

[27] The inequality $\boldsymbol{v} \leq \mathbb{E}\left[\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w} \mid \bar{\mathbf{u}}) \mid \tilde{\boldsymbol{w}}\right]$ should be taken element by element.

(a) for every $\tilde{w}_f$, we have that

$$\max\ v_f - v_m \qquad \textit{subject to} \quad (v_f, v_m) \in \partial \mathbb{E}\mathcal{U}_{\mathcal{M}}(\tilde{w}_f, \tilde{w}_m)$$

is infinity for every $\tilde{w}_m$ or is finite for every $\tilde{w}_m$.

(b) for every $\tilde{w}_m$, we have that

$$\max\ v_m - v_f \qquad \textit{subject to} \quad (v_f, v_m) \in \partial \mathbb{E}\mathcal{U}_{\mathcal{M}}(\tilde{w}_f, \tilde{w}_m)$$

is infinity for every $\tilde{w}_f$ or is finite for every $\tilde{w}_f$.

Proposition 14 (Galichon, Kominers, and Weber (2019))**.** *Under assumption B.1 there is a unique equilibrium to the marriage market. That is, for each $\tilde{\boldsymbol{w}}$ there is a disagreement utility function $\bar{\mathbf{u}}_{\tilde{\boldsymbol{w}}}$ such that there are unique utilities*[28]

$$\boldsymbol{v}_{\mathcal{M}}(\tilde{\boldsymbol{w}}) \leq \mathbb{E}[\boldsymbol{u}_{\mathcal{M}}(\boldsymbol{w} \mid \bar{\mathbf{u}}_{\tilde{\boldsymbol{w}}}) \mid \tilde{\boldsymbol{w}}], \tag{12}$$

which clear the marriage market,[29]

$$D_{f,\mathcal{M}}(\tilde{\boldsymbol{w}} \mid \boldsymbol{v}_{\mathcal{M}}) = D_{m,\mathcal{M}}(\tilde{\boldsymbol{w}} \mid \boldsymbol{v}_{\mathcal{M}}).$$

Remark. If the set of disagreement utility functions, $\bar{U}$, is too restrictive then the inequalities in (12) may be strict. For example, consider the case where $\bar{U}$ contains a single element; In this case, we have a non-transferable utility model (at the marriage stage). Similarly, some mechanisms may restrict transfers too much and preclude the existence of an equilibrium with equality in (12).

[28] Here $\leq$ should be taken element by element.

[29] $D_{f,\mathcal{M}}(\tilde{w}_f, \tilde{w}_m \mid \boldsymbol{v}) = \mathbb{P}\Big(v_f(\tilde{w}_f, \tilde{w}_m) + \varepsilon_f(\tilde{w}_m) = \max v_f(\tilde{w}_f, \tilde{w}'_m) + \varepsilon_f(\tilde{w}'_m)\Big)$, and similarly for $D_{m,\mathcal{M}}(\tilde{w}_f, \tilde{w}_m \mid \boldsymbol{v})$.

C Online Appendix: Computing the Optimal Tax System

C.1 Baseline

The baseline tax system is based on the federal tax system brackets for 2006. The tax rates for each bracket are calibrated to match the marginal tax rate in Heathcote et al. (2017) which approximates the statutory taxes prevalent in the period. The after-tax income schedule for each household in a group is

$$\text{ATI}(Y) = \lambda Y^{1-\rho}.$$

The tax rates and tax brackets we use are presented in Table 6.

tax rate (%)	-14.77	13.66	28.52	36.28	42.89	47.81
Single	0 - 7.55	7.55 - 30.65	30.65 - 74.20	74.20 - 154.8	154.8 - 336.55	336.55 - ∞
Married Jointly	0 - 15.10	15.10 - 61.30	61.30 - 123.70	123.70 - 188.45	188.45 - 336.55	336.55 - ∞
Married Separate	0 - 7.55	7.55 - 30.65	30.65 - 61.85	61.85 - 94.26	94.26 - 168.28	168.28 - ∞
Head-of-Household	0 - 10.75	10.75 - 41.05	41.05 - 106.00	106.00 - 171.65	171.65 - 336.55	336.55 - ∞

Table 6: Tax brackets (in \$1000) for each tax rate of our baseline tax system.

C.2 Cubic Spline Approximation

Given breakpoints $0 = \tau_1 < \cdots < \tau_n$, define the piecewise cubic function

$$P_i(x) = c_{1,i} + c_{2,i}(x - \tau_i) + c_{3,i}(x - \tau_i)^2 + c_{4,i}(x - \tau_i)^3 \qquad \tau_i \leq x \leq \tau_{i+1}$$

with

$$\begin{aligned} c_{1,i} &= P_i(\tau_i) = g(\tau_i) \\ c_{2,i} &= P_i'(\tau_i) = s_i \\ c_{3,i} &= \frac{P_i''(\tau_i)}{2} = \frac{[\tau_i, \tau_{i+1}]g - s_i}{\Delta\tau_i} - c_{4,i}\Delta\tau_i \\ c_{4,i} &= \frac{P_i'''(\tau_i)}{6} = \frac{s_i + s_{i+1} - 2[\tau_i, \tau_{i+1}]g}{\Delta\tau_i^2} \end{aligned}$$

This function interpolates the points $g(\tau_1), \ldots, g(\tau_n)$.

If we set for $i = 2, \ldots, n-1$

$$s_{i-1}\Delta\tau_i + s_i 2(\Delta\tau_{i-1} + \Delta\tau_i) + s_{i+1}\Delta\tau_{i-1} = b_i$$

with

$$b_i = 3(\Delta\tau_i[\tau_{i-1}, \tau_i]g + \Delta\tau_{i-1}[\tau_i, \tau_{i+1}]g),$$

for $i = 1$

$$s_1 \Delta\tau_2 + s_2(\Delta\tau_2 + \Delta\tau_1) = \frac{(\Delta\tau_1 + 2(\Delta\tau_1 + \Delta\tau_2))\Delta\tau_2[\tau_1, \tau_2]g + \Delta\tau_1^2[\tau_2, \tau_3]g}{\Delta\tau_2 + \Delta\tau_1},$$

and for $i = n$

$$s_{n-1}(\Delta\tau_{n-1} + \Delta\tau_{n-2}) + s_n \Delta\tau_{n-2} = \frac{\Delta\tau_{n-1}^2[\tau_{n-2}, \tau_{n-1}]g + (2(\Delta\tau_{n-1} + \Delta\tau_{n-2}) + \Delta\tau_{n-1})\Delta\tau_{n-2}[\tau_{n-1}, \tau_n]g}{\Delta\tau_{n-1} + \Delta\tau_{n-2}},$$

we have a cubic spline. De Boor and De Boor (1978)

We can rewrite in matrix notation,

$$\boldsymbol{M}\mathbf{s} = \mathbf{b}$$

where

$$\boldsymbol{M} = \begin{bmatrix} \Delta\tau_2 & \Delta\tau_2 + \Delta\tau_1 & 0 & 0 & \dots & 0 & 0 & 0 \\ \Delta\tau_2 & 2(\Delta\tau_2 + \Delta\tau_1) & \Delta\tau_1 & 0 & \dots & 0 & 0 & 0 \\ 0 & \Delta\tau_3 & 2(\Delta\tau_3 + \Delta\tau_2) & \Delta\tau_2 & \dots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \dots & 0 & \Delta\tau_{n-1} & 2(\Delta\tau_{n-2} + \Delta\tau_{n-1}) & \Delta\tau_{n-2} \\ 0 & 0 & 0 & \dots & 0 & 0 & \Delta\tau_{n-2} + \Delta\tau_{n-1} & \Delta\tau_{n-2} \end{bmatrix}$$

And

$$\mathbf{b} = \boldsymbol{HKD}\mathbf{g}$$

where

$$\boldsymbol{D}_{n-1\times n} = \begin{bmatrix} -1 & 1 & 0 & \dots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -1 & 1 \end{bmatrix},$$

$$\boldsymbol{H}_{n\times n-1} = \begin{bmatrix} \frac{(3\Delta\tau_1 + 2\Delta\tau_2)\Delta\tau_2}{\Delta\tau_2 + \Delta\tau_1} & \frac{\Delta\tau_1^2}{\Delta\tau_2 + \Delta\tau_1} & 0 & \dots & 0 \\ 3\Delta\tau_2 & 3\Delta\tau_1 & 0 & \dots & 0 \\ 0 & 3\Delta\tau_3 & 3\Delta\tau_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 3\Delta\tau_{n-1} & 3\Delta\tau_{n-2} \\ 0 & \dots & 0 & \frac{\Delta\tau_{n-1}^2}{\Delta\tau_{n-1} + \Delta\tau_{n-2}} & \frac{(3\Delta\tau_{n-1} + 2\Delta\tau_{n-2})\Delta\tau_{n-2}}{\Delta\tau_{n-1} + \Delta\tau_{n-2}} \end{bmatrix},$$

and

$$\boldsymbol{K}_{n-1\times n-1} = \mathrm{diag}(\Delta\tau_i^{-1}).$$

With this, we can compute

$$\mathbf{c}_1 = \boldsymbol{T}\mathbf{g} = \boldsymbol{C}_1\mathbf{g}$$

$$\mathbf{c}_2 = \boldsymbol{T}\mathbf{s} = \boldsymbol{T}\boldsymbol{M}^{-1}\boldsymbol{H}\boldsymbol{K}\boldsymbol{D}\mathbf{g} = \boldsymbol{C}_2\mathbf{g}$$

$$\mathbf{c}_3 = \boldsymbol{K}\left(3\mathbf{I}_{n-1} - (\boldsymbol{T}+\boldsymbol{R})\boldsymbol{M}^{-1}\boldsymbol{H}\right)\boldsymbol{K}\boldsymbol{D}\mathbf{g} = \boldsymbol{C}_3\mathbf{g}$$

$$\mathbf{c}_4 = \boldsymbol{K}^2\left(\boldsymbol{R}\boldsymbol{M}^{-1}\boldsymbol{H} - 2\mathbf{I}_{n-1}\right)\boldsymbol{K}\boldsymbol{D}\mathbf{g} = \boldsymbol{C}_4\mathbf{g}$$

where

$$\boldsymbol{R}_{n-1\times n} = \begin{bmatrix} 1 & 1 & 0 & \dots & 0 \\ 0 & 1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 & 1 \end{bmatrix}$$

and

$$\boldsymbol{T}_{n-1\times n} = \begin{bmatrix} \mathbf{I}_{n-1} & \mathbf{0}_{n-1\times 1} \end{bmatrix}$$

Two-dimensional tensor product cubic spline Consider two-dimensional breakpoints $\{\tau_1, \dots, \tau_n\} \times \{\tau_1, \dots, \tau_n\}$, indexed by i and j, which interpolates the points

$$\mathbf{G}^{\mathrm{T}}_{n\times n} = \begin{bmatrix} \mathbf{g}_1, \dots, \mathbf{g}_n \end{bmatrix}.$$

For each j, define the coefficients

$$\mathbf{c}_j = \boldsymbol{C}\mathbf{g}_j$$

where

$$\boldsymbol{C}_{4(n-1)\times n} = \begin{bmatrix} \boldsymbol{C}_4 \\ \boldsymbol{C}_3 \\ \boldsymbol{C}_2 \\ \boldsymbol{C}_1 \end{bmatrix}$$

Then,

$$\mathbf{c}_{4(n-1)\times n} = \begin{bmatrix} \mathbf{c}_1, \dots, \mathbf{c}_n \end{bmatrix} = \boldsymbol{C}\mathbf{G}^{\mathrm{T}}$$

Finally, the two-dimensional spline coefficients are given by

$$\boldsymbol{C}\mathbf{G}\boldsymbol{C}^{\mathrm{T}}$$

Therefore, we can compute the derivative as

$$(\boldsymbol{C} \otimes \boldsymbol{C})\mathrm{vec}(D\mathbf{G})$$

Define $\boldsymbol{CC} = (\boldsymbol{C} \otimes \boldsymbol{C})$.

Derivative of utility with respect to tax schedule If we approximate the after-tax income by selecting $\mathbf{G}$ values to interpolate at income breakpoints $\{\tau_1, \ldots, \tau_n\} \times \{\tau_1, \ldots, \tau_n\}$, we can compute the derivative of the after-tax income at z_f, z_m in agreement by following the steps

1. Map (z_f, z_m) to the breakpoints by

$$\tau_i \leq z_f < \tau_{i+1}, \qquad \tau_j \leq z_m < \tau_{j+1}$$

2. From the matrix of coefficients, the relevant terms are

$$((n-1)(k_1-1)+i, (n-1)(k_2-1)+j), \qquad \text{for} \quad k_1, k_2 \in \{1,2,3,4\},$$

which are associated to the polynomial terms

$$(z_f - \tau_i)^{4-k_1}(z_m - \tau_k)^{4-k_2}$$

3. Alternatively (and more relevantly for the derivative), we can write

$$\mathrm{idx}_{k_1,k_2} = (n-1)(k_1-1) + i + 4(n-1)((n-1)(k_2-1) + j - 1)$$

or, we can write

$$\mathbf{idx} = \begin{bmatrix} \mathrm{idx}_{1,1}, & \mathrm{idx}_{1,2}, & \ldots & \mathrm{idx}_{k_1,1}, & \mathrm{idx}_{k_1,2}, & \ldots & \mathrm{idx}_{4,3}, & \mathrm{idx}_{4,4} \end{bmatrix}^{\mathrm{T}}$$

that is, we read the coefficients row-by-row.

4. Then we can find the derivative of ATI with respect to a change in G as

$$\sum_{k_1,k_2=1}^{4} (z_f - \tau_i)^{4-k_1}(z_m - \tau_j)^{4-k_j}\boldsymbol{CC}(\mathrm{idx}_{k_1,k_2}, :)\mathrm{vec}(D\mathbf{G})$$

or

$$(z_f - \tau_i)^{4-\mathbf{k}_1} \cdot (z_m - \tau_j)^{4-\mathbf{k}_2}\boldsymbol{CC}(\mathbf{idx}, :)\mathrm{vec}(D\mathbf{G})$$

where

$$\mathbf{k}_1^{\mathrm{T}} = \begin{bmatrix}1\\2\\3\\4\end{bmatrix} \otimes \begin{bmatrix}1\\1\\1\\1\end{bmatrix} \qquad \text{and} \qquad \mathbf{k}_2^{\mathrm{T}} = \begin{bmatrix}1\\1\\1\\1\end{bmatrix} \otimes \begin{bmatrix}1\\2\\3\\4\end{bmatrix},$$

5. therefore we have that

$$ATI(z_f, z_m) = (z_f - \tau_i)^{4-\mathbf{k}_1} \cdot (z_m - \tau_j)^{4-\mathbf{k}_2} \boldsymbol{CC}(\mathbf{idx}, :)$$

and

$$\partial_f ATI(z_f, z_m) = (4 - \mathbf{k}_1) \cdot (z_f - \tau_i)^{3-\mathbf{k}_1} \cdot (z_m - \tau_j)^{4-\mathbf{k}_2} \boldsymbol{CC}(\mathbf{idx}, :)$$

$$\partial_m ATI(z_f, z_m) = (4 - \mathbf{k}_2) \cdot (z_f - \tau_i)^{4-\mathbf{k}_1} \cdot (z_m - \tau_j)^{3-\mathbf{k}_2} \boldsymbol{CC}(\mathbf{idx}, :)$$

From piecewise polynomial (PP) to B-spline The PP (piecewise-polynomial) representation above is better to evaluate derivatives. However, the B-spline representation has advantages when we want to impose shape constraints on our spline.

From the PP-spline $P_i(x)$ in the previous section, we can rewrite it in the B-form by defining the sequence of knots $t_1 \leq t_2 \leq \cdots \leq t_{n+6}$,

$$t_1 = t_2 = t_3 = t_4 = \tau_1 < \tau_2 = t_5 < \cdots < \tau_{n-1} = t_{n+2} < \tau_n = t_{n+3} = t_{n+4} = t_{n+5} = t_{n+6}$$

Define $\mathbf{t} = \{t_1, \ldots, t_{n+6}\}$.

and coefficients

$$\alpha_i = P_i(t_{i+2}) + \frac{\Delta t_{i+2} - \Delta t_{i+1}}{3} P_i'(t_{i+2}) - \frac{\Delta t_{i+2} \Delta t_{i+1}}{6} P_i''(t_{i+2})$$

Then, there is a unique set of functions $B_{i,4,\mathbf{t}}$, the spline basis functions, such that De Boor and De Boor (1978)

$$P_j(x) = \sum_{i=j-3}^{j} \alpha_i B_{i,4,\mathbf{t}}(x), \qquad t_j \leq x \leq t_{j+1} \quad \text{and} \quad j \in \{4, \ldots, n+2\}$$

Therefore, given the coefficients $\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3$, and $\mathbf{c}_4$ extended to define $c_{k,n}$ as well,[30] we can find the coefficients $\boldsymbol{\alpha}$ by

$$\boldsymbol{\alpha} = A_1 \mathbf{c}_1 + A_2 \mathbf{c}_2 + A_3 \mathbf{c}_3 + A_4 \mathbf{c}_4,$$

[30]That is,

$$c_{1,n} = g_n \quad c_{2,n} = s_n \quad c_{3,n} = c_{3,n-1} + 3c_{4,n-1}\Delta\tau_{n-1} \quad c_{4,n} = c_{4,n-1}.$$

where

$$A_1 = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \ddots & \vdots & \vdots \\ \vdots & 0 & \ddots & 0 & \vdots \\ \vdots & \vdots & \ddots & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{e}_1^{\mathrm{T}} \\ \mathbf{I}_n \\ \mathbf{e}_n^{\mathrm{T}} \end{bmatrix}.$$

$$A_2 = \frac{1}{3} \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ \Delta\tau_1 & 0 & \dots & 0 & 0 \\ 0 & \Delta\tau_2 - \Delta\tau_1 & \ddots & \vdots & \vdots \\ \vdots & 0 & \ddots & 0 & \vdots \\ \vdots & \vdots & \ddots & \Delta\tau_{n-1} - \Delta\tau_{n-2} & 0 \\ 0 & 0 & \dots & 0 & -\Delta\tau_{n-1} \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{0}^{\mathrm{T}} \\ \Delta\tau_1 \mathbf{e}_1^{\mathrm{T}} \\ \boldsymbol{D}_{n-2\times n-1} \Delta\boldsymbol{\tau} \\ -\Delta\tau_{n-1} \mathbf{e}_n^{\mathrm{T}} \\ \mathbf{0}^{\mathrm{T}} \end{bmatrix}$$

$$A_3 = -\frac{1}{3} \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ 0 & \Delta\tau_2 \Delta\tau_1 & \ddots & \vdots & \vdots \\ \vdots & 0 & \ddots & 0 & \vdots \\ \vdots & \vdots & \ddots & \Delta\tau_{n-1} \Delta\tau_{n-2} & 0 \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix}$$

$$A_4 = \mathbf{0}$$

Putting everything together, define

$$\boldsymbol{A}_{(n+2)\times 4n} = \begin{bmatrix} A_4 & A_3 & A_2 & A_1 \end{bmatrix}$$

then, for the one-dimensional case, we have

$$\boldsymbol{\alpha} = \boldsymbol{A} \widetilde{\boldsymbol{C}} \mathbf{g},$$

and for the two-dimensional case, we have

$$\boldsymbol{\alpha} = \boldsymbol{A}\widetilde{\boldsymbol{C}}\mathbf{G}\widetilde{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{A}^{\mathrm{T}}$$

and

$$\mathrm{vec}(\boldsymbol{\alpha}) = (\boldsymbol{A}\widetilde{\boldsymbol{C}}) \otimes (\boldsymbol{A}\widetilde{\boldsymbol{C}})\mathrm{vec}(\mathbf{G})$$

If the after-tax income is written as cubic B-spline (tensor) with knots $\{t_i\}$, then

$$ATI(z_f, z_m) = \sum_{i,j} \alpha_{i,j} B_{i,k}(z_f) B_{j,k}(z_m)$$

with $k = 4$ (the spline order).

Then, the derivative is given by

$$\partial_f ATI(z_f, z_m) = \sum_{i,j} \alpha_{i,j}(k-1) \left(\frac{B_{i,k-1}(z_f)}{t_{i+k-1} - t_i} - \frac{B_{i,k-1}(z_f)}{t_{i+k-1} - t_i} \right) B_{j,k}(z_m),$$

which can be rearranged as

$$\partial_f ATI(z_f, z_m) = (k-1) \sum_{i,j} \left(\frac{\alpha_{i,j} - \alpha_{i-1,j}}{t_{i+k-1} - t_i} \right) B_{i,k-1}(z_f) B_{j,k}(z_m).$$

This implies that we can force monotonicity by imposing

$$\alpha_{i,j} - \alpha_{i-1,j} \geq 0 \quad \forall i, j,$$

and, by changing the roles of i and j,

$$\alpha_{i,j} - \alpha_{i,j-1} \geq 0 \quad \forall i, j.$$

Imposing that the function is concave is harder (the B-spline representation does not help here). However, it is simple to enforce concavity in each dimension separately.

By applying the above reasoning again, we have that

$$\partial_{ff} ATI(z_f, z_m) = (k-1)(k-2) \sum_{i,j} \frac{1}{t_{i+k-2} - t_i} \left(\frac{\alpha_{i,j} - \alpha_{i-1,j}}{t_{i+k-1} - t_i} - \frac{\alpha_{i-1,j} - \alpha_{i-2,j}}{t_{i+k-2} - t_{i-1}} \right) B_{i,k-2}(z_f) B_{j,k}(z_m)$$

We can impose concavity (or progressivity) on each dimension separately by imposing that

$$\frac{\alpha_{i,j} - \alpha_{i-1,j}}{t_{i+k-1} - t_i} - \frac{\alpha_{i-1,j} - \alpha_{i-2,j}}{t_{i+k-2} - t_{i-1}} \leq 0 \quad \forall i, j,$$

and

$$\frac{\alpha_{i,j} - \alpha_{i,j-1}}{t_{j+k-1} - t_j} - \frac{\alpha_{i,j-1} - \alpha_{i,j-2}}{t_{j+k-2} - t_{j-1}} \leq 0 \quad \forall i, j.$$

Using that $k = 4$, and defining $\Delta^3 t_i = t_{i+3} - t_i$ we have

$$\Delta^3 t_{i-1} \alpha_{i,j} - (\Delta^3 t_{i-1} + \Delta^3 t_i)\alpha_{i-1,j} + \Delta^3 t_i \alpha_{i-3,j} \leq 0 \quad \forall i, j,$$

and

$$\Delta^3 t_{j-1} \alpha_{i,j} - (\Delta^3 t_{j-1} + \Delta^3 t_j)\alpha_{i,j-1} + \Delta^3 t_j \alpha_{i,j-3} \leq 0 \quad \forall i, j$$